Scaling Up SimCalc Project

Design and Development of Curriculum Units and Professional Development

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Jennifer Knudsen

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Research collaborators:

University of Massachusetts, Dartmouth; Virginia Polytechnic Institute and State University;
The University of Texas at Austin; and the Charles A. Dana Center at the University of Texas at Austin

SRI International
Center for Technology in Learning
333 Ravenswood Avenue
Menlo Park, CA 94025-3493
650.859.2000

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Prepared by:

Jennifer Knudsen, SRI International

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Design and Development of Curriculum Units and Professional Development

The SimCalc research team conducted two experiments to test whether a combination of professional development and SimCalc-based curriculum materials could be used by a wide variety of teachers to support their students' learning of conceptually complex mathematics. The experiments were conducted with about 150 seventh- and eighth-grade teachers from across the state of Texas. This technical report describes the content of the units, their features, the training that accompanied the units and issues in its development.

Unit Content

Two curriculum units were designed for these experiments, one for seventh-graders and one for eighth-graders. Each of the curriculum units addresses Texas state standards and also includes topics that were more challenging than those in the standards. *Managing the Soccer Team* addresses seventh-grade standards on rate and proportionality and includes multi-rate functions and the meaning of slope. *Designing Cell Phone Games* targets eighth-grade standards for linear function and introduces average rate through further exploration of multi-rate functions. A detailed description of each unit—the storyline, the activities, the main mathematical ideas, and the learning goals—is appended. What follows is a more compact description.

Both units follow a similar progression. Each begins with simple analyses of motion at a constant speed, and follows a learning progression that culminates in the more complex topics. Moving from qualitative to quantitative analyses, the unit develops graphical, tabular and symbolic forms of linear functions. In the seventh grade units, the functions are directly proportional, so that each point represents the same rate of change. In the eighth grade units, proportional and non-proportional functions are distinguished. Each of these topics is found in the Texas state standards. The progression then continues in each unit to address multi-rate functions—piecewise linear functions. The eighth grade unit continues on to explore average rate. Each of these topics is developed in contexts that can be represented in the SimCalc MathWorlds®' "world" window, a place where motion or growth can be shown with characters and other representations of real-world phenomena. Motion and non-motion contexts are important in both units: both to help students generalize across contexts and to provide another opportunity to learn complex content.

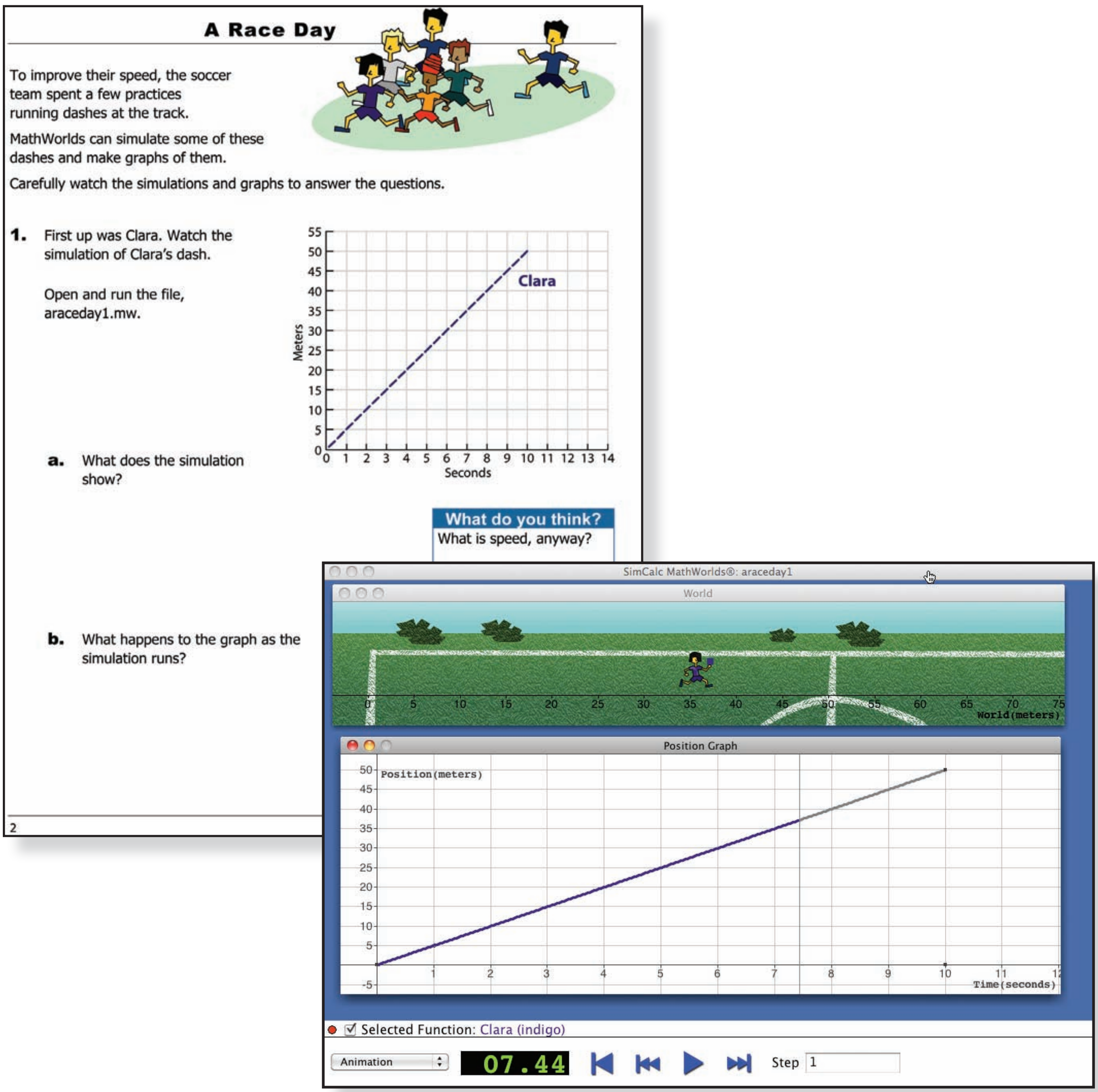
A tour through *Managing the Soccer Team* will illustrate one of the progressions in some detail:

> Early in the soccer team unit, students observe a simple line graph and a simulation of a soccer player running along a straight line (Exhibit 1). The graph and simulation are linked so that as the runner moves forward, the graph of that motion builds. In the graph window, the x-axis represents time and the y-axis represents distance traveled. At first, students just identify the starting and ending time of the runner, correlating the runner's position in the world window and the graph window. A series of questions in the student workbook guides

students from this initial interpretation of the graph to the understanding that the entire line represents the speed of the runner, the ratio of distance to time.

In the middle of the unit, *Run, Jace, Run* introduces symbolic algebraic notation to represent the same type of motion (Exhibit 2). Students express the relationship between time and distance first in a graph, then in a table, then in words, and finally in an equation. Both table and language representations of motion enable students to write the equation of the line. Through more activities, students develop facility in connecting representations and translating from one to another.

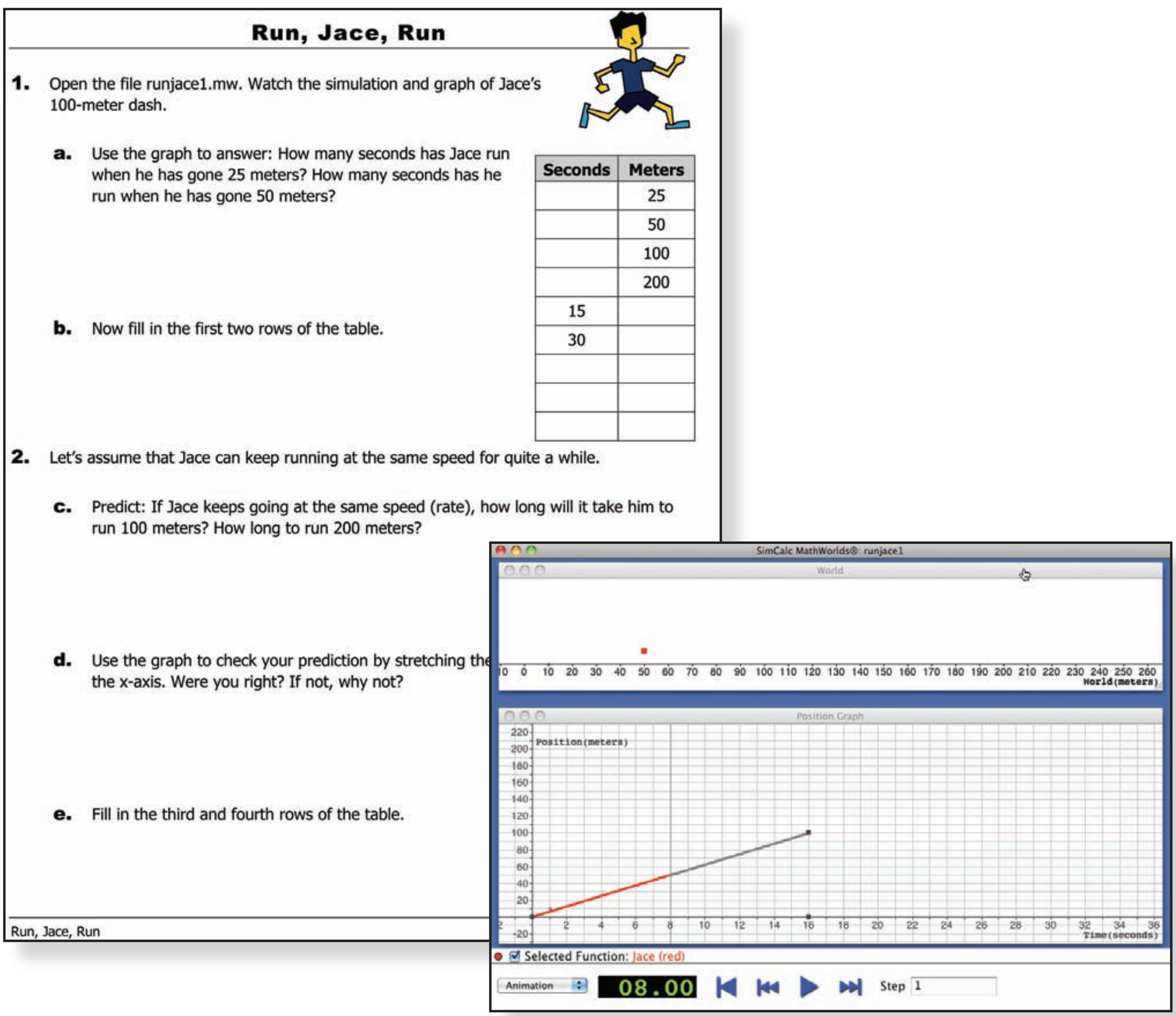
Exhibit 1. Activity from Early in the Unit



Back to the Office is an activity that parallels *Run, Jace, Run:* it presents the same content in a non-motion context. Students solve problems involving, for example, best buys on soccer cones and setting ticket prices for the team's games. *Back to the Office* re-introduces content in a non-motion context, allowing students to generalize but also to learn what they may have missed the first time around. There are a number of these activities throughout the unit.

On the Road consists of a series of stories about the troubled history of the team's trips from its hometown to Dallas. In each trip, a bus and a van traveled between the two cities, stopping, slowing and speeding up—and sometimes turning around and going the other way. The activity allows students to build correspondences between multi-rate, piecewise linear graphs and verbal descriptions of motion. Slope is dealt with qualitatively. This activity is where the unit starts to move beyond state standards for seventh grade. But the unit's storyline and contexts remain familiar, as does the core concept of rate.

Exhibit 2. Activity Midway through the Soccer Unit



Run, Jace, Run

1. Open the file runjace1.mw. Watch the simulation and graph of Jace's 100-meter dash.

a. Use the graph to answer: How many seconds has Jace run when he has gone 25 meters? How many seconds has he run when he has gone 50 meters?

b. Now fill in the first two rows of the table.

Seconds	Meters
	25
	50
	100
	200
15	
30	

2. Let's assume that Jace can keep running at the same speed for quite a while.

c. Predict: If Jace keeps going at the same speed (rate), how long will it take him to run 100 meters? How long to run 200 meters?

d. Use the graph to check your prediction by stretching the the x-axis. Were you right? If not, why not?

e. Fill in the third and fourth rows of the table.

Run, Jace, Run

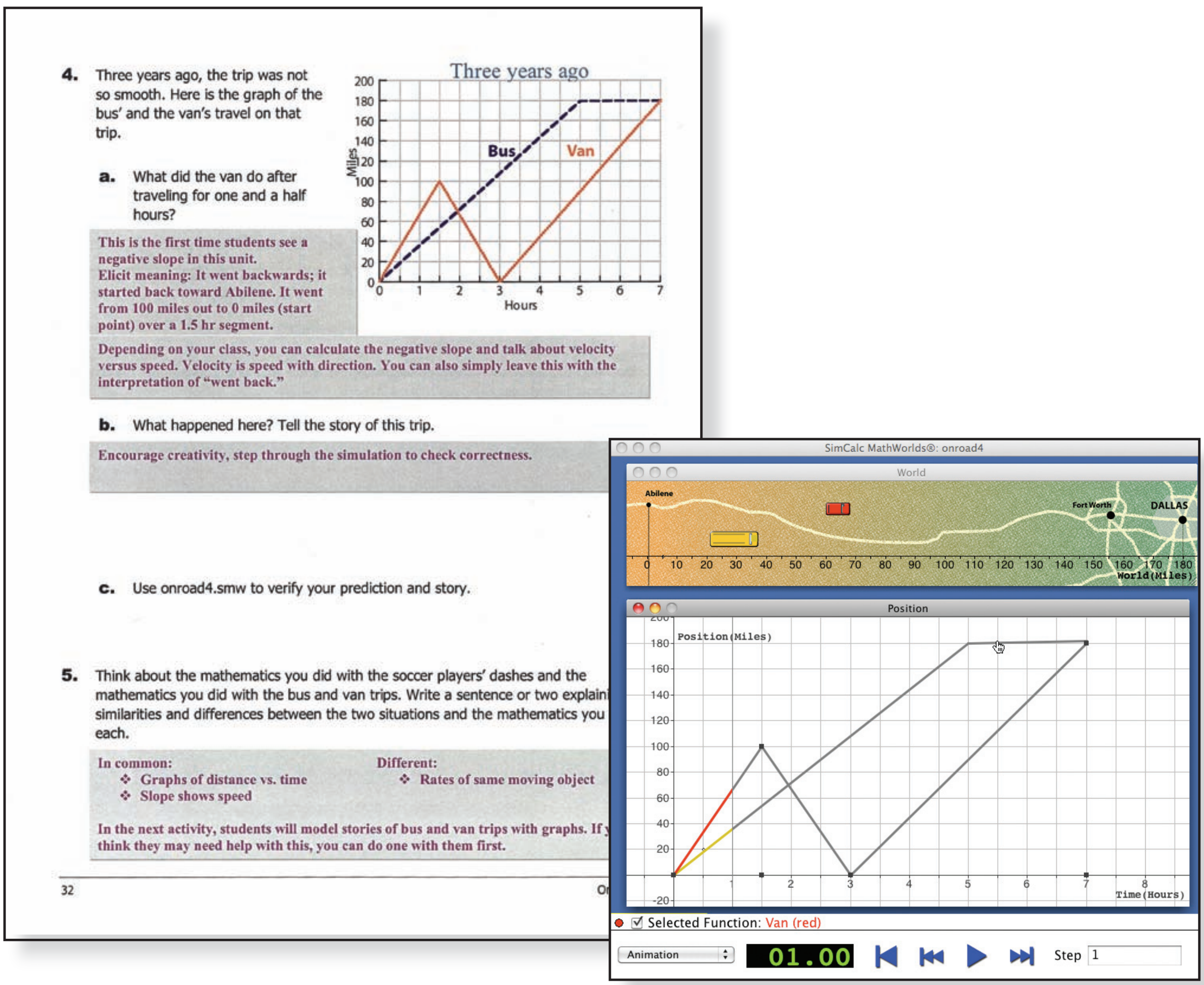
Unit Features

In addition to detailing the mathematics learning progression, the unit designers used a set of features to support teachers and students in their use of the unit.

Both the teacher notes and the student materials were designed to be helpful to teachers in planning and implementing each unit. Teachers who might never open their teacher notes could instead use the student workbook as a kind of default guide or lesson plan. Important questions for the class to consider are written into the student book—in fact, every important question that the developers could think of was treated this way. On the other hand, the workbook is not a prescriptive script—teachers were free to write their own lesson plans, augmenting or highlighting different parts of the student workbook. A third option was to use the teachers' notes to adapt premade day-by-day lesson plans. These plans provided extra questions to ask students, and included sample student responses. A suggested pacing chart helped teachers plan for how to complete the unit in whatever amount of time they might have—with the recommended time being 10 days.

Other features of the units were designed to help students directly. SimCalc MathWorlds® software allows animated versions of objects and people in motion in contexts. The theme of each unit provides continuity across these contexts and the storyline provides detail that the software could not capture.

Exhibit 3. Activity from Second Half of Unit



Realistic numbers are used so that students could check speeds, prices and other rates against their knowledge of how expensive uniforms are or how fast people run. The text uses simple sentence structure and consistent vocabulary, never going beyond a fifth-grade reading level, in order to accommodate those with low-level reading skills and those just learning to read English in making sense of the context and the math. To help guide and organize students' work, the workbook uses graphical conventions to indicate various kinds of activities and content. For example, definitions appeared inside boxes on the page, as did other critical content information. The amount of white space left after a question indicated the type and length of an expected answer. Simple graphics served as implicit indices for the activities. Even the fact that the workbook contained all the student activities physically bound together provided another organizational aid to students. The workbooks contained as much color as the budget would allow, appealing to media-savvy students who are used to plentiful use of color.

Professional Development

The seventh-grade and eighth-grade experiments used two different professional development (PD) models. The first was designed to best ensure that the researchers' and developers' intentions were reflected in the PD and therefore reached teachers. The second was designed to be better aligned with professional development practices in Texas. The workshops offered in the PD were less than a week long, about as much as could be expected from teachers for training in teaching a replacement unit.

For Managing the Soccer Team, the goals of the 5 days of training were to

- Provide teachers with a mental image of the unit as a whole, as well as direct experience with most activities.
- Improve teachers' content knowledge, both in terms of the mathematics in the unit—going deeper—and the mathematics that might come after the unit—going farther.
- Allow teachers to develop comfort with the software
- Model teaching strategies consistent with a SimCalc approach.
- Show how the unit fit in with local customs and addressed state standards.

The workshop had three parts. In the first part, a well-known local expert in math professional development led the teachers in transitioning from a view of proportionality as $a/b = c/d$—a relationship among four numbers—to a broader view of proportionality as a linear function of the form $y = kx$—relating an infinite number of pairs of numbers. This understanding was key to considering rate and proportionality in the SimCalc context, where functions are the primary object of study. Additionally, this transition was advocated by influential state education groups; materials similar to theirs were used in the workshops. For the next two days, a member of the SRI team led teachers through the Managing the Soccer Team unit, using their workbooks and the software. The leader modeled a small set of SimCalc strategies such as "predict, check, explain," in which teachers were encouraged to predict what would happen in SimCalc MathWorlds® simulations before running them and then explain the differences between what they predicted and what actually happened. To boost teachers' comfort with the software, the leader gave demonstrations and encouraged teachers to play with the software, as well as use it in the unit activities. In the third part of the workshop, teachers used additional activities and the software to explore calculus concepts, learning about the relationship between graphs of velocity and speed. Teachers learned this more advanced mathematics core to SimCalc, both so that they could understand where the mathematics in the soccer unit could lead and so that they could experience new content, in the same way that Managing the Soccer Team is likely to be for their students.

The eighth grade experiment used a two-tiered system of training—a "train the trainers" approach. The same pair of professional developers led a 2-day training for local professional development experts, who then delivered a 3-day training session for teachers in the regions of Texas that they served. These local experts were provided with curriculum, teacher notes and presentation slides. They were asked to do a training much like that described for the second two parts of the seventh-grade study.

Balancing Constraints

Every curriculum is designed within constraints—which might be about the people, the resources or the politics in a given situation. Designing the Scaling Up SimCalc units was no different—but with the added constraint of the experimental setting in which the materials were used.

Designing for the experiment and for other constraints was a balancing act. The units, of course, needed to well represent the SimCalc approach to teaching and learning—that is what the experiments were designed to test. Typically, SimCalc is used to help young students, middle through high school, learn the fundamental ideas of calculus by providing real-life interpretations for differentiation and integration (hence the same SimCalc). But these topics were more advanced that those in the middle school mathematics standards in Texas, where the experiments took place. The teachers, administrators and policymakers associated with the experiment wanted to ensure that materials used by thousands of students would address the most important topics in their standards. Finding a balance between typical SimCalc topics and state standards-based content resulted in the content described in this report. By inching past the standards, with multi-rate functions, we managed to stay within the comfort zone of local education leaders as well as many teachers, while still pushing into the "precalculus" territory of SimCalc.

Another part of the experimental design was recruiting a wide variety of teachers in terms of background, experience, pedagogical style, and mathematics knowledge. In designing the teacher supports described, this wide variety was considered. The materials needed to meet the needs of teachers who struggled with the math, without over-constraining teachers with advanced content knowledge. The teachers with advanced content knowledge, though, might be still developing pedagogical skills, and so both the skillful questioner and the teacher struggling to keep order in class needed to be supported. Finally, other work had revealed that many of today's teachers do little written planning before delivering their lessons. So the training included time and structure to help teachers do that planning, an important part of teaching particularly for novice teachers.

Pedagogy is often embedded in curriculum and made explicit during training. Traditionally, SimCalc materials presented fairly complex and open-ended problem situations in which students could gradually develop mathematical insights. This was not the commonly endorsed pedagogy at the time, and not many teachers would have had exposure to such methods. So the experiment's units were more highly structured than customary SimCalc materials, and the PD leaders did not push teachers to adopt wholly new practices.

Conclusion

This report described several aspects of the curriculum and professional development used in two scaling up experiments. Paper materials and software served to guide students in an exploration of real-world contexts and associated mathematics representations, focusing on rate, proportionality, and linear function. Developers of the unit took into account not only the mathematics that could be learned using a SimCalc approach, but also state standards that were essential to address. Teachers were supported in their classroom use of the materials through a set of teacher notes and professional development that focused on teachers' mathematics learning and effective implementation of the unit. The developers took into consideration then-current state standards and assessments in their curriculum and PD design.

Appendix A
Math Content in the Managing the Soccer Team and Designing Cell Phone Games

Table A1. Managing the Soccer Team

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
1	• Managing the Soccer Team • A Race Day • Another Race Day • Info Quest	In a graph of objects in motion at a constant speed in the coordinate plane, the x-axis typically represents time; the y-axis, distance; and the speed can be found from any point on the line relating time and distance.	1. Find the speed of an object moving at a constant rate, starting at 0 time and 0 distance, by calculating from the endpoints and later from any point on the line. 2. Identify the x-axis, y-axis, line graph, and given meaning for them in specific context.	Students are appointed as temporary team managers when the current manager mysteriously leaves town. The first task is to time players making dashes, straight-line runs designed to improve players' speed. Students also find information about the real-life speed of runners and other objects and creatures.
2	• Isabella Improves • Faster Than Max • Practice Runs	For graphs of objects in motion at a constant speed (following the conventions above): If distance is constant, as the amount of time decreases, the speed increases. Steeper lines represent faster speeds.	1. Understand relationships among speed, time, and distance, as represented by graphs and numbers or described in words. 2. Describe patterns in a graph. 3. Given a time/distance graph of the motion of two objects (single rate only each), determine the distance between objects at a given time. 4. Have a qualitative understanding that greater slopes (informally, steeper) represent faster motion, in time/distance graphs. 5. Draw time/distance graphs for two moving objects (each at a constant rate), given sufficient information. (e.g.; time, end points, relative times)	Students explore the relationships among lines of dashes of different speeds and duration.

Table A1. Managing the Soccer Team (Continued)

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
3	• Run, Jace, Run • Run, Jace, Run, Revisited	Given an object moving at a constant speed and a graph and table representing related distances and times, entries in the table can be found given the numbers in two rows. Each row in the table can be used to calculate the speed. Every point on the associated line graph can be used to calculate the constant speed/steepness of the line. From the table or the graph, it is possible to write an equation that is also a representation of the constant speed and that can be used to find the distance at any time. In previous lessons, speed could be calculated using the time and distance at the endpoint of a graph. Now speed is a rate that relates each time to a distance and can be expressed as an equation with two variables.	1. Assuming a constant speed (proportional relationship), complete entries in a time/distance table by a variety of valid methods reasoning across and down table columns. 2. Use the table to write an equation that relates time to distance. 3. Identify graphs, equations, and tables representing the same proportional relationship.	One student's running is used to explore and unite tables, equations, and graphs.
4	• Back At the Office	The same ideas as in lesson 2 are developed in the context of money and objects.	1. Assuming a constant rate relating two variables (proportional relationship), complete entries in a table by a variety of valid methods reasoning across and down table columns. 2. Use the table to write an equation that relates the two variables. 3. Describe a situation that could be represented by a given equation (using discrete, nonmotion context). 4. Identify graphs, equations, and tables representing the same proportional relationship.	Managing the team includes off-field activities, too. This lesson uses problems about buying soccer cones and uniforms, for example, to develop the same ideas of proportionality and functions as in the earlier activities. This also generalizes the ideas presented earlier; they apply in more than motion contexts.

Table A1. Managing the Soccer Team (Continued)

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
5	• Slope and Rate	The definitions of rate, unit rate, and slope can be connected to the previous contexts and representations. A traditional way to calculate slope is by creating triangles based on rise/run as related to a line graph.	1. Understand and use definition of unit rate. 2. Connect unit rate to slope. Understand slope as a numerical description of the relationship between any two points on a line.	This activity is a pure mathematics time-out to give mathematical vocabulary for the concepts explored more informally earlier.
6	• On the Road • Road Trips	Graphs with connected line segments at different slopes can represent an object moving at different speeds (with x-axis as time, y as distance) • Positive slope represents moving ahead (in the positive direction.) • Negative slope represents moving backward (in the negative direction.) • When the slope is 0, the represented object is not moving. • The length of the 0 slope line segment gives the period of time the object is standing still.	1. Interpret graphs representing moving objects with two or more rates; give contextual story associated with each graph. Given story, create graph. 2. Make these connections/ interpretations: • Positive slope means moving ahead. • Negative slope means going back. • 0 slope means standing still. • The length of the 0 slope line indicates how long standing still.	The team has a mysteriously bad history of making trips to the state championships. The trips are taken with both a bus and a van. Students explore and create stories that match graphs representing these trips.
7	• Graphs and Motion	No new mathematics content This activity provides practice in motion contexts, including single- and multiple-rate motions and the coordination of graphs and narratives. It also helps students learn to interpret “on sight” certain typical multirate linear graphs.	Recognize typical graph shapes for: • Standing still • Going one rate, then going at a faster rate for the same amount of time • Going one rate, then going at a slower rate for the same amount of time • Going forward and then backward at the same rate	

Table A1. Managing the Soccer Team (Continued)

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
8 - 9	• Salary Negotiations • Summer Job Advice • All About MPG • How Far On ow Much? MPG • Suiting Up	No new mathematics concepts. Exercises and problem solving use concepts introduced earlier.		Students have done so well as interim manager that they are offered the job, with two choices of salary. After they decide which choice is best, they then advise other students who are facing similar issues in making summer job choices.
10	• Mathematically Speaking	If you can write a formula $y = kx$ where k is a real number and y and x represent varying quantities, with a single y value for each x, then we can say that the quantities x and y vary proportionally. A unit rate tells how many y you get for one x.	Use the definitions of unit rate and proportionality to identify these concepts in the curriculum.	

Table A2. Designing Cell Phone Games

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
1	• Working at TexStar Games • Cell Phone Games and Design • Yari, the Yellow School Bus • Our First Cell Phone Game	Motion can be represented on a time/distance graph. Lines can represent idealized motion (without changes in speed).	1. Find the distance that an object moves, its speed, and for how much time it travels given a time/distance graph of that motion represented as a line. 2. Calculate speed from time and distance—from the endpoints of the graph and from any point on the graph (beginning level). 3. Compare the graph of the motion of an object "idealized" as a line with a graph that is more accurate.	Students are hired at the cell phone design firm TexStar Games to use math to improve the business. The students use math to design how the games would work.

Table A2. Designing Cell Phone Games (Continued)

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
2	• Controlling Characters with Graphs	Graphs can be mathematical representations of motion, from which students can compare positions, speeds, and starting places of moving objects. For a position/time graph, steeper lines represent faster speeds.	1. Find speed using multiple methods in a graph—unit rate, “slanty-ness” (informal version of slope), and end points. 2. Use math knowledge to change the speed of objects in the software by changing the graph. 3. Understand the connection between informal slope and speed. 4. Interpret word problems about speeds, start, and end times using graphs.	Students use graphs to control the speeds of cars in designing a road rally game.
3–4	• Controlling Characters with Equations • One to Another • Better Games	Equations are another form of mathematical representation used to represent functions (word used only informally), as are tables. The equation $y = kx + b$ is a common form for writing linear functions, where y and x are variables and k and b are constants. If x is time and y is distance, k usually represents speed and b is the starting point of the objects. Tables can help us write equations. One can translate among graphs, tables, and equations.	1. Write equations of the form $y = kx$ for moving objects starting from a fixed origin (time 0, position 0). 2. Write equations of the form $y = kx + b$ for moving objects that start at different non-zero positions 3. Complete tables, equations, and graphs based on sufficient information across all three.	Students use equations to control the motion of characters in cell phone games, including those with three characters.
5	• Wendella’s Journey	Multisegment graphs can represent an object moving at different speeds. You can tell a story to explain the motion of an object over time to match such a graph. Flat (informal slope 0) lines represent standing still. Lines that are slanted downward (informal version of negative slope) represent moving backward.	1. Create graphs that represent forward motion at slower and faster speeds, backward motion and no motion. 2. Create stories that match such graphs. Create graphs that match stories about motion.	Students create journeys for a game character that is a dog featured in a game about motion.

Table A2. Designing Cell Phone Games (Continued)

Lesson	Activities	Main mathematical ideas	Goals (students will be able to)	Story line
6	• Money Matters	Multisegment graphs can be used to show other kinds of accumulation over time, such as money in a bank account.	1. Interpret multisegment graphs in a time/money situation. 2. Predict using multisegment graphs.	Students help TexStar understand its cash balance.
7	• Mathematically Speaking: Graphs to Know • Crab Velocity	Velocity is speed with direction. Negative rates indicate backward motion. Position can be negative, where 0 is a defined starting point that objects can be on either side of.	1. Sketch graphs that show, for example, increasing and then decreasing rates of motion, increasing and then decreasing rates of savings. 2. Represent backward motion with negative rates and do this in equations.	Students use equations and graphs to control crabs in a game in which crabs move above and below the waterline.
8	• Wolf and Red Riding Hood	No matter how characters move, if their motion graphs have the same endpoint then they meet at the same time in the same place.	1. Make qualitative predictions of a single rate that gets an object to the same place at the same time as an object that is moving at two different rates. 2. Use equations to represent a single rate that gets an object to the same place at the same time as an object that is moving at two different rates.	Students analyze a game in which players try to keep Red Riding Hood from being eaten by a wolf.
9	• Secrets of Average Rate Revealed • Problem Solving • Problems from the TexStar Lunchroom	Average rate is the single rate that could “stand in” for the rates to be averaged; it would accomplish the same thing. For example, if an object is traveling at several rates then the average rate is a single rate that will get the object to the same place at the same time.	1. Understand the connection between rates and their average. 2. Solve problems involving average rates—to find average rates, missing rates, and to account for different amounts of time for various rates in calculating their average. 3. Use graphs to estimate or calculate average rates.	Students solve problems from the bulletin board and conversation in the lunchroom at work.
10	• Linear Relationships: Proportional and Non-propor-tional • TexStar Games: Going Full Time	Proportional relationships can be expressed as $y = kx$, linear relationships can be expressed as $y = kx + b$. Therefore, proportional relationships are linear relationships where $b = 0$. When b is not = to 0, then the linear relationship is non-proportional. All functions that are not linear are also non-proportional.	Distinguish between proportional and non-proportional equations using graphs or equations or from situations expressed in words.	Students explain to TexStar why the work they have done in the past 2 weeks qualifies them for a position; both mathematical and nonmathematical work are included.

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SRI International
Center for Technology in Learning
333 Ravenswood Avenue
Menlo Park, CA 94025-3493
650.859.2000

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