Designing Cell Phone Games

An 8th-grade unit on linear functions

Teacher Edition

Designing Cell Phone Games

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The project is a collaboration among

Center for Technology in Learning
SRI International
Principal Investigator: Jeremy Roschelle

The Charles A. Dana Center at the University of Texas, Austin
Co-Principal Investigator: Bill Hopkins

University of Massachusetts, Dartmouth
Co-Principal Investigator: Stephen Hegedus

The University of Texas at Austin
Co-Principal Investigator: Susan Empson

Virginia Polytechnic Institute and State University
Co-Principal Investigator: Deborah Tatar

Writing and Editing: Jennifer Knudsen, Ken Rafanan, Phil Vahey, Gucci Estrella
Editing and Document Management: Ken Rafanan, Gucci Estrella, Rebecca Moniz, Meredith Ittner
Design: Ken Rafanan, Lynne Peck Theis
Editorial Advice: Jeremy Roschelle, Deborah Tatar, Bill Hopkins, Jim Kaput
This unit is inspired by and based on the work and ideas of Jim Kaput.

Contents

Unit Overview

This unit focuses on linear functions. It places them in a context of proportion and nonproportional functions. It shows how linear functions can be used to model situations and solve problems involving a constant rate of change: from motion to money. Informal uses of slope concepts are introduced to compare rates visually (steeper /faster connection, for example). It also explores methods for writing equations based on situations, tables and graphs and the connection among them.

TEKS Addressed

(1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 8 are using basic principles of algebra to analyze and represent proportional and nonproportional relationships and using probability to describe data and make predictions.

(2) Throughout mathematics in Grades 6-8, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use concepts, algorithms, and properties of rational numbers to explore mathematical relationships and to describe increasingly complex situations. Students use algebraic thinking to describe how a change in one quantity in a relationship results in a change in the other; and they connect verbal, numeric, graphic, and symbolic representations of relationships.

(8.3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

- (A) compare and contrast proportional and nonproportional relationships; and
- (B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.

(8.4) **Patterns, relationships, and algebraic thinking.** The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

- (A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equation...

(8.2) **Number, operation, and quantitative reasoning.** The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

- (D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, $a = 1.4h$.

Materials

Student workbooks

Computers, at least one for every 2-3 students, with SimCalc software installed. Internet access is not required.

Computer with large display for teacher

Straight edges

Helpful: colored pencils, graph paper, calculators

Computer Use

Whole class discussions.. 1 computer with large display visible to all

Group Work.. 1 computer for every 2-3 students

Homework.. Dependent on activity

The ideal is to have a set of computers in your classroom (laptops are ok) that you can set up for groupwork and put aside during whole class discussions. A computer lab can be used.

Before the first class, run the software (SimCalc Mathworlds) as if you were a student. Take note of how long it takes to load the program and where students can locate the user files. If you want students to save their work, give them instructions on where they can store their files.

General Teaching Tips

- Encourage explanations of answers. When students give number answers, ask, "How do you know that?"
- Promote inquiry. Encourage other students to ask questions of each other when another explanation does not make sense.
- Balance whole class work with individual and group work. Students need both. Sometimes it helps to start with open-ended work where they have to figure out what to do. But, if they don't know how to start AT ALL, some whole class work first can help.
- Let students do more talking than you do. Even in whole class mode, let students answer the questions and challenge the answers others give.
- Make sure you specify the time for whole class discussion so you don't have to try to pry kids away from using the computers.
- Allocate additional time for students to become familiar with the MathWorlds software during the first class.
- Adjust your pace to the needs of your students. The pacing guides presented in subsequent pages do not take into consideration specific needs and constraints in some classrooms.

Using this Teacher Guide

- Suggested lesson plan pages precede student activity pages.
- Student activity pages have hand-written answers and are keyed to the lesson plan.
- Extension activities are also included in some notes.

Lesson Plan Day 1

Activities		
Working at TexStar Games	Whole class discussion	10 minutes
Cell Phone Games and Design	Whole class discussion	
Yari, the Yellow School Bus	Group work	15 minutes
Our First Cell phone Game	Group work	25 minutes

Big Ideas

- Context of the unit: the mechanics and business of cell phone game design, such as how to design components of a game and computing salary and savings.
- Motion can be represented on a graph of time vs distance.
- Idealized motion on a time-distance graph appears as a straight line. (HINT: This is a stepping stone to averages.)
- "Steeper" lines represent higher speeds.

Notes

Working at TexStar Games

Cell phone Games and Design

1. Have students read the text aloud.
2. Ask questions to make sure they understand the basic context.
3. Explain that in this unit, we do the math to set up the games. We won't see the final game, but students should imagine how it might work. Students also learn about how and how much money game designers make.
4. Wherever possible, the numbers in this unit are realistic. We got them from real designers and design companies.
5. If possible, show a real cell phone game in the "moving character" genre. This is the kind of game we are developing in this unit.
6. Introduce the SimCalc software.
   a. Walk students through opening the student files and saving their work, if applicable.
   b. Point out the 4 windows available: simulation, table, graph and equation.
   c. Remind them that the sheet with quick commands is in their workbook.

Yari, the Yellow School Bus

1. Ask students: If you give a wheeled toy a push across the room, when will it go fastest? Slowest?
2. After watching the movie, elicit "changing speed."
3. Have students graph a number of points using the *stop action* buttons on the movie.

Our First Cell phone Game

1. Emphasize: Rita, the new character, is a SIMULATION of the real toy in the movie. We simplify the motion so that we can work with line graphs.
2. For Question 1: Students should make the connection between total distance, total time and speed. This is probably familiar territory.
3. For Question 2: Focus on the whole line, i.e., set of points. Get a qualitative explanation.
   a. Standard: "As the graph builds, the y axis shows the same number as the "ruler" in the simulation."
   b. More sophisticated, optional: "the line is slanting up because as the bus moves along both time and distance get bigger (are increasing.)"
4. For Question 3: Using the table, build correspondence between overall speed and the rate distance/time at EVERY point on the line. This begins building the idea of constant rate.

Working at TexStar Games

Welcome to the headquarters of TexStar Games, Inc.

We make games for cell phones—teens love them!

Other game design companies are cutting into our business so we need to make improvements. That's why you have been hired.

To make TexStar Games more competitive you will

- Use mathematics to analyze our game designs; for example, determine how to make characters move across the screen.
- Analyze our business.

This will require a lot of mathematics. Now's the time to learn it! With the materials in this book, you will learn about functions, both proportional and nonproportional. You will also learn the "real truth" about average rates. And you will understand the connections between tables, graphs, and algebraic expressions. Don't worry, it will all be clear by the time you are done!

All this math will help us in our business, but you will also use it in later math classes and in your life. Keep that in mind as we move ahead.

Cell Phone Games and Design

A few facts about cell phone game design will help you in this unit.

Electronic games—on cell phones, on consoles, on computers—all must be designed (created) by a team of people. The team works together, but there are different jobs to do.

Designers come up with ideas for games and rules for how the game will work. A designer must think about how realistic the game is and what rules will make the game fun and just difficult enough. Math is used in doing this.

Programmers take the ideas and turn them into games. They use mathematics and logic to write programs.

Artists make the images you see on the phone as you play. They must use math to think about how big to make the images on the screen and how much room their art will take up.

Business staff make sure the business is running smoothly. They are in charge of things like employee salaries and tracking how much money the company is saving and spending.

Cell phone designers, programmers, and artists get paid for their work in different ways. Sometimes all these people work for a company like TexStar Games, and they get a paycheck every month. Other times, the programmers, designers, and artists work on their own and get paid for each piece of work they do.

The cell phone business is growing, and it's likely that more designers, programmers and artists will be needed in the future. Knowing math helps.

Yari, The Yellow School Bus

TexStar Games is working with a traditional windup toy company to design new games for younger kids. We need to analyze the motion of the real toy. Then we can make our cell phone version. Our partner's best selling toy is Yari, The Yellow School Bus.

1. Watch the movie of the actual windup toy —Yari, The Yellow School Bus—moving along a centimeter ruler. Open the file *Yari.swf*. Run it several times. Answer the following:

   A. How many centimeters (cm) did Yari travel? 100 cm

   B. How many seconds did Yari travel? 10 seconds

> **Explain**
> Each time you give a number answer, explain how you know.

   C. Using A & B, calculate Yari's speed (how fast Yari traveled) for this trip. $\frac{100 \text{ cm}}{10 \text{ seconds}} = \frac{10 \text{ cm}}{1 \text{ seconds}}$

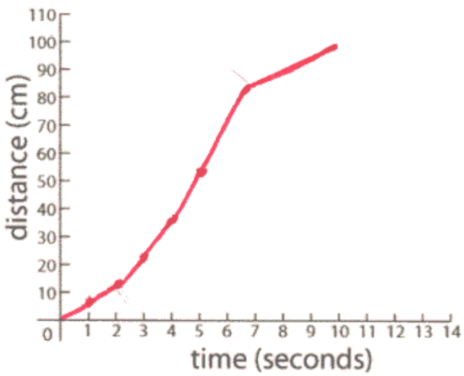
   D. Explain how your answers for A, B, and C are related.

2A. In the movie, when did Yari go *fastest*? (circle your answer.)
Beginning (Middle) End

2B. In the movie, when did Yari go *slowest*? (circle your answer.)
Beginning Middle (End)

3. Using the movie and stop action, fill in the table and make the graph.

seconds	cm
1	5
2	12
3	23
4	37
5	54
7	83
10	100



Our First Cell Phone Game

Our programmer has made our first cell phone game. It has a picture of Rita, The Red School Bus, which is used in the game.

We want to compare the motion of the classic windup toy Yari to our character in the cell phone game named Rita, The Red School Bus.

1. Open the Java MathWorlds file named *FirstGame1.smw*. Use the play button to watch the trip of the character in the simulation and the graph. Run the simulation as many times as you need to answer the following.

   A. How many centimeters (cm) did Rita travel? 100 cm

   B. How many seconds did Rita travel? 10 seconds

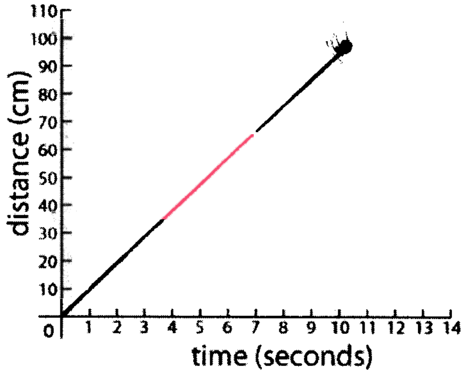
   C. What was the Rita's speed? 10 cm per seconds

2. Using step and play, watch how the graph builds as the character moves. Explain how the graph and the simulation are related.

   When the graph shows a specific distance, that's how far Rita has gone in the simulation.

3. Complete the table below:

seconds	cm
1	10
2	20
3	30
4	40
5	50
7	70
10	100



4. Describe how

A. the trips of the cell phone character and the real windup toy are *the same.*

They travel the same distance in
same amount of time.

B. the trips of the cell phone character and the real windup toy are *different.*

the cell phone character moves at the same speed
all throughout.

The real windup toy speeds up ÷ slows down

For Your Information
In mathematics, we often simplify things so that we can work with them. Two ways we simplify are using averages and creating models. Rita is a model of the classic windup toy, Yari. The cell phone game character is a *model* of the real windup toy. It simplifies the motion of the windup toy. Models always simplify the "real thing" in some ways.

Lesson Plan Day 2

Activities

Controlling Characters with Graphs: Texas Road Rally	Group work	50 minutes

Big Ideas

- Graphs are mathematical representations of relationships such as motion.
- Graphs of motion show characters' start position, speed (relative) and places where characters meet.
- For graphs of motion, i.e., time v. distance: the steeper the line, the faster the motion.

Notes

Introduction

- *Road Rally Game*—cars move across a map of Texas, city to city, as in a real road rally. Explain that we are making the graphs to control the motion of the cars, setting up the math that will make the game work.

Question 1 (5 minutes)

- Help students connect distance/time and "slant of line" as two ways to find the speed of a car.

Question 2 (20 minutes)

- Axes labeled for students.
  - Sketch graph means draw the basic shapes. Showing the relative slants of the lines is the important part.
  - Speed is position/ time—needs units to be meaningful. Miles per hour, etc.

Question 3 (20 minutes)

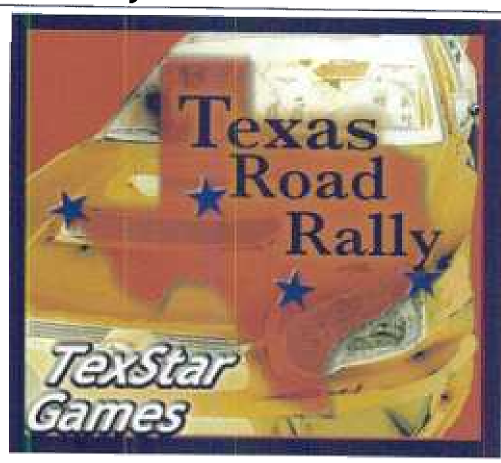
- Have students draw graphs in workbooks first, then check in SimCalc. (20 minutes)
  - Each problem has many solutions.
  - Can establish (informal) foundation for slope and y-intercept, and intersection of lines.
  - Can skip E if needed; parallel lines not essential to unit.

EXTENSION: Ask about what parts of a car's motion we can predict by looking at its time vs. distance graph. This summarizes and sets up the next lesson on equations. (5 minutes)

Controlling Characters with Graphs: Texas Road Rally

We need to set up the mathematics controllers for the new game, *Texas Road Rally*. We can use graphs to control motion. Remember: we'll make the game cool later!

In road rally races, cars compete as they travel from town to town.



For Your Information

Graphs, tables and equations are mathematical representations. Each can show the same motion in a different way.

1. Open *Roadrally1.smw*. It shows the Yellow Rose team car in the Texas Road Rally. The car starts in El Paso and stops in Big Spring. Run the file, watching the simulation and graph.

   A. When the car stops, how far has it traveled and how much time has passed?

   350 miles , 7 hours

   B. When the car has traveled for one hour, how far has it gone?

   50 miles

   C. How fast is this car going? Explain how you know in two ways. Think about your answers to A and B or use other ideas.

   50 miles per hour

   ① Reading from the graph, you can see that the car traveled 50 miles in 1hour.

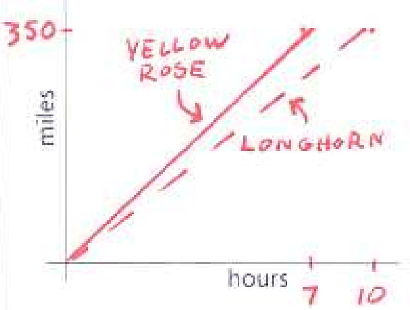
   ② $\frac{350 \text{ miles}}{7 \text{ hours}} = \frac{50 \text{ miles}}{1 \text{ hour}} = 50 \text{ miles/hr}$ or 50 mph

   D. Change the speed of the car using the graph window. It still must start in El Paso and stop in Big Spring. Use the same ideas to find the new speed. What is the new speed?

   < Answers will vary but the graph should start at 0 and end at 350 miles >

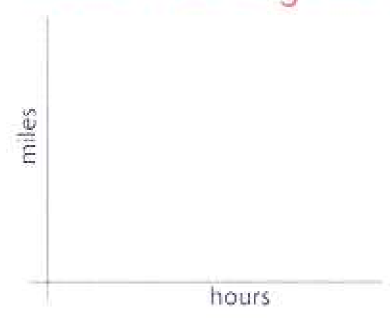
2. Open but DO NOT RUN *Roadrally2.smw*. It shows graphs of the Yellow Rose team car and the Longhorn team car ready to travel from San Antonio to Beaumont.

   A. Sketch the graph of the two cars' trips on the same axes and fill in the boxes.

	Speed of Each Car	Fastest Car
	YELLOW ROSE : 50 mph LONGHORN: 35 mph	Yellow Rose

   B. Use the graph window to change the speed of each car so that the other car is faster this time. Now fill in the same information as in A.

< Answers will vary but the slope of Longhorn should be steeper >

	Speed of Each Car	Fastest Car
		Longhorn

   C. By looking at the position/time graphs of two cars in a rally, how can you predict which one will win?

   The car with the steeper line wins.

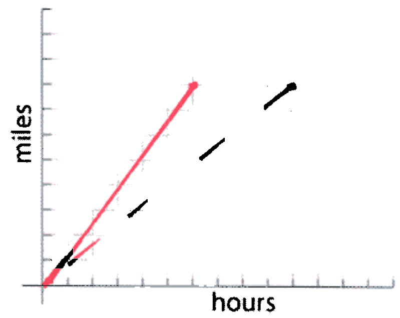
   D. Explain how the cars' speeds and their graph lines are related to one another.

   The steeper the line, the faster the car.

3. Below are ideas to use in other games. Please help our programmers by drawing graphs on the axes provided. Test out your graph using *Roadrallytest3.smw.* Make up your own characters for the Green Iguana and Blue Sky team cars. *Note: There is more than one way to draw the graph for most of these.*

   A. Draw a graph of a race in which the Green Iguana and Blue Sky team cars start at the same position and travel the same distance but at different speeds.

< Sample Answer >



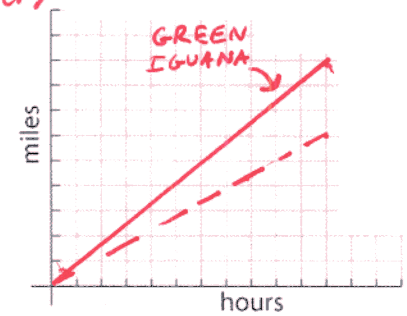
CRITICAL FEATURES:

✓ Both endpoints have the same y-axis value (x, y)

✓ different steepness

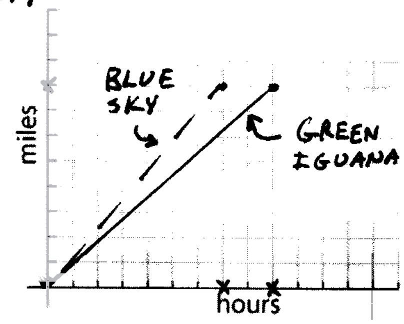
   B. Draw a graph of a race in which the two characters start at the same position and travel the same amount of time, but Green Iguana is faster than Blue Sky.

< Sample Answer >



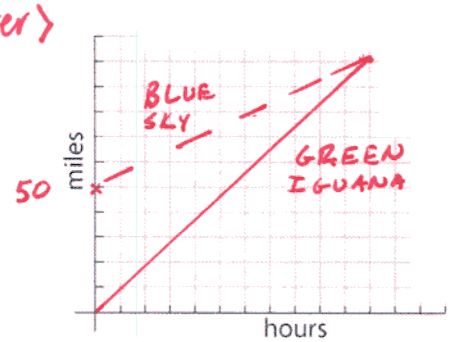
C. Draw a graph of a race in which the two characters start at the same position and travel the same distance, but Blue Sky finishes two hours before Green Iguana.

<Sample Answer>



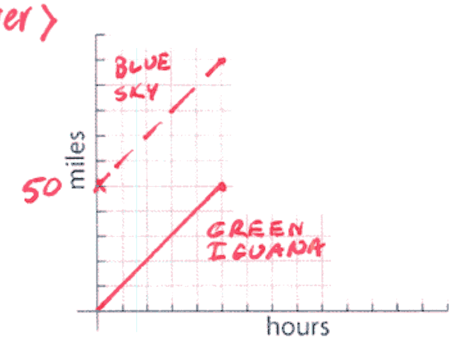
D. Draw a graph of a race in which Green Iguana starts at the starting line (0 miles), and Blue Sky starts 50 miles ahead (50 miles). The two characters finish at the same position and the same time.

<Sample Answer>



E. Draw a graph of a race in which Green Iguana starts at the starting line and Blue Sky starts 50 miles ahead. The two characters travel at the same speed and travel the same distance.

<Sample Answer>



CRITICAL FEATURES:

√ Label y-axis

√ Lines should be parallel

Lesson Plan Day 3

Activities		
Controlling Characters with Equations	Group work	50 minutes
One to Another	Homework/ Group work	

Big Ideas

- Equations, also known as algebraic expressions, are another form of mathematical representation. Graphs and tables are other forms.
- Equations can be written based on tables. (The process will be presented in this activity
- You can "translate" between graphs, tables and algebraic expressions.

Notes

Controlling Characters with Equations

Introduction

Robot game: players use remote-control robots to open doors, detonate bombs, etc.

Question 1 (10 minutes)

- Accept all answers; this sets up the process they learn next.
- Students should observe the connection between the table, graph and equation.
- Encourage students to make their best guess on how they can write equations themselves.

Question 2 (25 minutes)

- Guide students as they discover how to write equations from tables.
  - Identify a pattern in the table, relating each pair in a row.
  - Write the rule numerically for several rows.
  - Use variable names to express the pattern algebraically.

☆ IMPORTANT NOTE: the method presented above is based on the assumption that the function represented is a linear one. It doesn't "go crazy" for numbers not on the table.

Question 3 (10 minutes)

- Students practice the method just developed.
- You can develop more practice items by having students change the lines in the file, gliderhal.mw

Question 4 (5 minutes)

- Summarize the method for writing equations from tables.
- Ask kids to identify what K represents and the process that the equation represents.
- Remind students: $y = Kx$.

Question 5 (Give as homework if there is not enough time.)

- Use this as a challenge or practice. Apply previously learned methods to problem solving.

EXTENSION: Have students draw corresponding graphs and pictures for

- time and position relationship
- non-motion relationship such as money in the bank, expressed as dollars over time

One to Another (homework or in class)

Practice for "translating" between graphs, tables and equations.

Lesson Plan Day 4

Activities

Controlling Characters with Equations (question 5)	Homework review	10 minutes
One to Another	Homework review	
Better Games	Group work	40 minutes

Big Ideas

- Representations can be "translated" from one to another: word, graphs, tables, equations.
- y=kx+b, where $b \neq 0$ is the commonly recognized notation for linear functions with non-zero intercepts.
- In motion contexts, b is typically the starting point.

Notes

Controlling Characters with Equations, question 5 and One to Another

- Review homework, i.e., question 5 of *Controlling Characters with Equations* and *One to Another*, this by having students' share their solutions and explain why they think they are right. (10 minutes)

Better Games

Introduction

- This is another Robot game. Retro-arcade means the robots move in a straight line, facing obstacles.
- Using the same methods as in the previous lessons, students develop equations for linear functions with non-zero intercept.

Questions 1-2

- Help students connect characters' start position with the number at which the line crosses y-axis. (Y-intercept, but unit does not use the term)

Question 3

- Help students develop process for writing questions of the form y=mx+b
- Link this to processes in earlier lessons.

Question 4

- Students connect graphs and equations directly.
- It's OK to have students make tables to help, if needed.

Controlling Characters with Equations

So far, our graphs of moving characters have shown the relationship between time and position—where the character is at what time.

Equations show that same relationship with numbers and symbols. This means: give us any time, and we can find the position of the character at that time.

We need to be able to control our characters with graphs as well as equations (also called algebraic expressions). The equations are so efficient!

Let's work on a game with robots. We need to set up the mathematics to make our robot move at different speeds.

1. Open *Shakey.smw*, which shows Shakey the Robot.

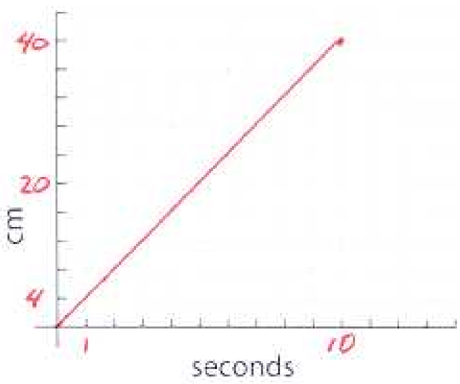
   A. Run the simulation. How fast is Shakey going? How do you know?

   4 cm per second

   Shakey travels 40 cm in 10 seconds

B. Look at the table, the graph, and the algebraic expression window. Record the graph, table, and equation below.

Graph



Table

x (seconds)	y (cm)
0	0
1	4
2	8
3	12
4	16
5	20
10	40

Equation

$y = 4x$

C. To answer these questions, run the *Shakey.smw* file using step as much as you need to.

- How is time represented in the table? In the equation?

  In the table, time is in seconds.

  In the equation, time is x.

- How is position represented in the table? In the equation?

  In the table, position is in cm.

  In the equation, position is y.

- How is speed represented in the table? In the equation?

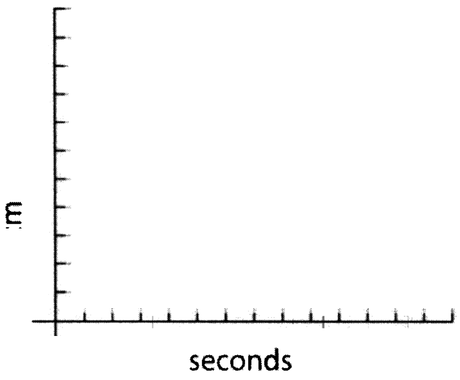
  In the table, speed is represented by the relationship between position & time.

  In the equation, speed is the number before x.

D. Change the graph so that Shakey moves slowly. Write down the table and equation for each graph. Then make Shakey move fast and write down the new information. < ANSWERS WILL VARY >

Slow Shakey

Graph



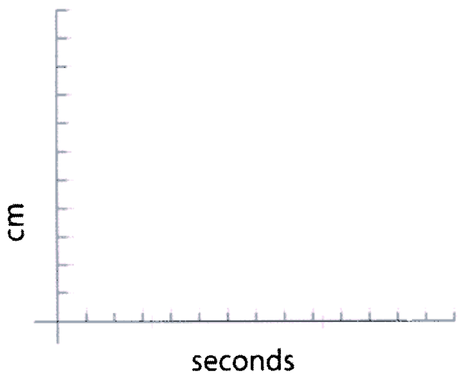
Table

x (seconds)	y (cm)
0	
1	
2	
3	
4	
5	
10	

Equation

Fast Shakey

Graph



Table

x (seconds)	y (cm)
0	
1	
2	
3	
4	
5	
10	

Equation

E. Describe how to write an equation if you have the table and graph that go with it. Use the tables, graphs, and equations above to help you.
If you are not sure yet, just make your best guess.

< ANSWERS WILL VARY >

2. Let's try another way to learn how to write equations from tables. No more algebra and table windows to help! Open *Roberta.smw*.

Variables and Letters
Here we use t and p as the variables in our equations instead of x and y.

A. First, fill in the p column in the table below.
Fill in the mathematical rule to get p, when you know t.

Roberta

t (time in seconds)	Rule to get p	p (position in cm)
0	0 x 8	0
1	|	8
2	|	16
3	|	24
4	|	32
5	|	40
10	|	80
t	t x 8	p

1st: Describe the pattern in the time column.

2nd: Describe the pattern in the position column.

3rd: Use those patterns to find the relationship that connects the time and the position in each row.

B. For every second that Roberta travels, how many centimeters does the robot move?

8 cm

At what rate does Roberta move?

8 cm per second

How does this rate relate to your rule?

To get p, multiply t times 8

Describe the pattern in the position column.

The column increases in multiples of 8

Use those patterns to find the relationship that connects the time and the position in each row.

For every 1 that t increases, p increases by 8

C. Write the equation that will work for any row that could be in the table.

$p = 8t$

3. Open *Gliderhal.smw*. Go through the same steps as in Question 2 to find the equation for these two robots' motion.

Glider

t (time in seconds)	Rule to get p	p (position in cm)
0	0 x 20	0
1		20
2		40
3		60
4		80
5		100
10		200
t	t x 20	p

1st: Describe the pattern in the time column.

2nd: Describe the pattern in the position column.

3rd: Use those patterns to find the relationship that connects the time and the position in each row.

Equation: $p = 20t$

Hal

t (time in seconds)	Rule to get p	p (position in cm)
0	0 x 12.5	0
1		12.5
2		25.0
3		37.5
4		50.0
5		62.5
10		125.0
t	t x 12.5	p

1st: Describe the pattern in the time column.

2nd: Describe the pattern in the position column.

3rd: Use those patterns to find the relationship that connects the time and the position in each row.

Equation: $p = 12.5t$

4. Explain, in your own words, how to use a table to find an equation that represents a character's motion.

<SAMPLE ANSWER> Find the rule that relates t to p
Then write the rule in symbols using $p = \_t$

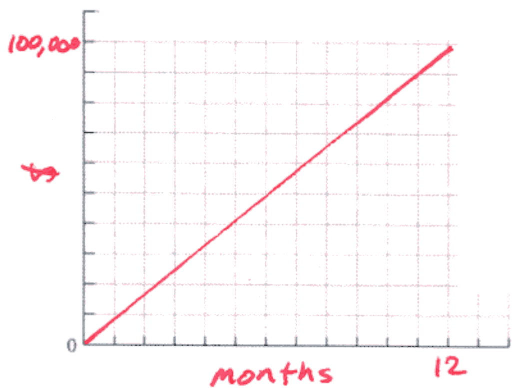
5. Use what you have learned to help Monica figure out the relationship between her salary and the time (number of months) she works.

> Monica is one of TexStar's best programmers. Every month, she earns a salary of $8,200. The programming team took the weekend off—they only left us a blank table to use. So, you have to do most of the work this time.

Make a representation (graph, table, or equation) so that she can choose any month (where January is month 1) and find out how much she has earned so far that year.

Equation: $S = 8{,}200m$

Months		$ earned
1		8,200
2		16,400
3		24,600
4		32,800
5		41,000
6		49,200
7		57,400
8		65,600
9		73,800
10		82,000
11		90,200
12		98,400



One to Another

You have seen how graphs, equations and tables can each represent a character moving at a certain speed. Now complete all the representations for each speed or rate. The axes of each graph are labeled for you.

You can use any variable when writing equations. In "Controlling Characters with Equations," we used t for time and p for position. In most, we used x and y. Name your own variable for each situation below. Just make sure that you use the same variables in your table and equation.

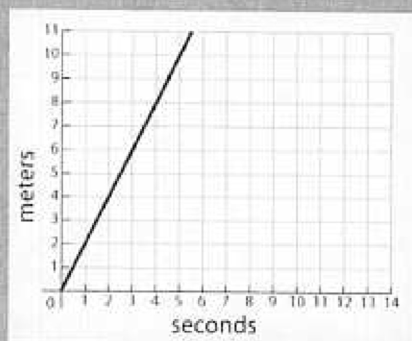
Example:

Gonzalo runs at a constant speed of 2 meters every second.

Table

x (seconds)	y (meters)
0	0
1	2
2	4
3	6
5	10

Graph



Equation

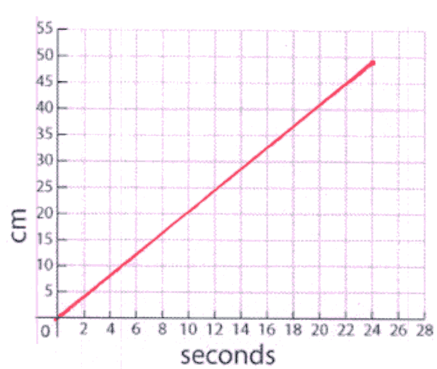
$$y = 2x$$

1. A toy robot moves on average 4 centimeters every 2 seconds.

Table

x (seconds)	y (cm)
0	0
24	48

Graph



Equation

$$y = 2x$$

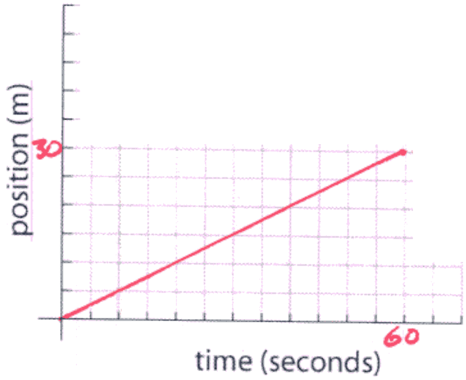
<TABLE VALUES & VARIABLE NAMES WILL VARY>

2. A sloth moves on average .5 meters every 1 second.

Table

t (seconds)	p (m)
0	0
60	30

Graph



Equation

$p = .5\,t$

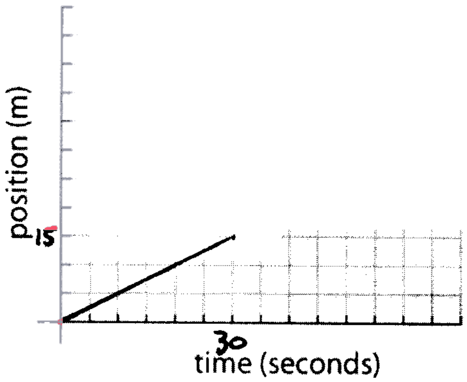
< TABLE VALUES WILL VARY >

3. An Australian Tiger Beetle runs on average 2.5 meters every 5 seconds.

Table

x (seconds)	y (m)
0	0
30	15

Graph



Equation

$y = 1/2\,x$

< TABLE VALUES & VARIABLE NAMES WILL VARY >

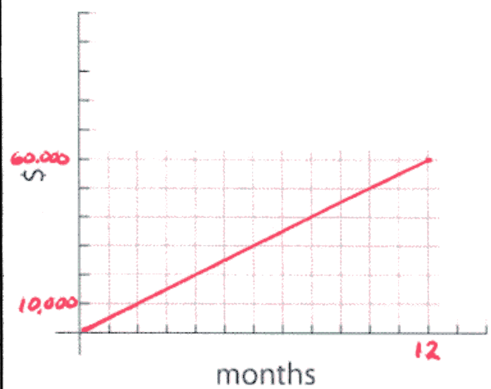
4. Instead of speed—or rate of motion—we can make the same representations for other rates, such as money earned per month.

A. Sergei, a beginning game designer at our company, earns an average of $5,000 per month.

Table

x (months)	y ($)
0	0
12	60,000

Graph



Equation

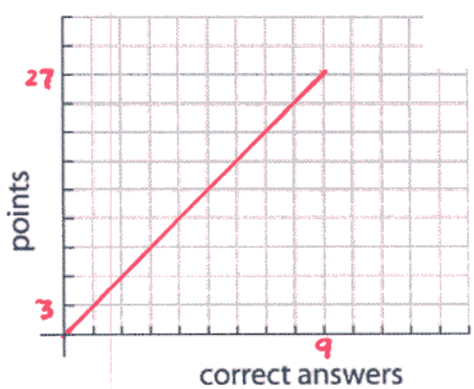
$y = 5{,}000x$

B. In a quiz game, players get 3 points for every correct answer.

Table

a (correct answers)	p (points)
0	0
9	27

Graph



Equation

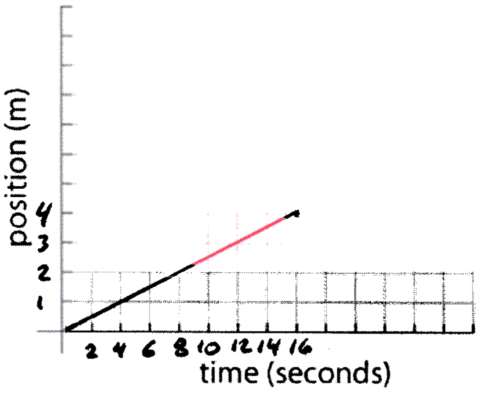
$p = 3a$

5. A toy car moves 1/4 meters every second.

Table

x (seconds)	y (m)
0	0
16	4

Graph



Equation

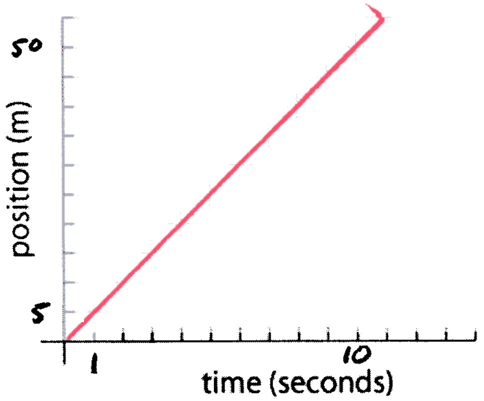
$y = 1/4\,x$

6. A helicopter flies up into the sky at 5 meters per second.

Table

x (seconds)	y (m)
0	0
15	75

Graph



Equation

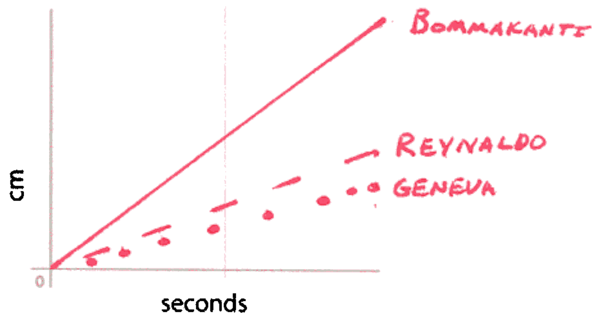
$y = 5x$

Better Games

We need to make mathematical controls for Reynaldo, Bommakanti, and Geneva, who will be used in a retro arcade game for cell phones. The equations are more complicated because these characters all start at different places.

Open *Better1.smw*.

1. Run the simulation. Predict what the graph will look like for each character's motion, by sketching three lines on the axes below. Don't forget to label your lines.



2. Now open the graph window. Run the file several times. Pay attention to the graph and the simulation and how they work together.

   A. Was your prediction right? Explain why or why not.

   < ANSWERS WILL VARY >

   B. Where did each character begin its motion? How does the graph represent this?

   Bommakanti : 0 cm ; Reynaldo : 5 cm ; Geneva = 10 cm

   The graph shows where each character begins at time 0

   C. What is the speed of each character?

   Bommakanti : 8 cm per sec ; Reynaldo : 6 cm per sec ; Geneva : 4.5 cm per sec

   Which character was fastest? Bommakanti

   Which character was second fastest? Reynaldo

   Which character was slowest? Geneva

   D. How did you know which character was fastest?

   the steeper line represents the faster character

3. You can find the equations for these characters' movements, using the method we learned in "Controlling Characters with Equations."

A. Let's start with Reynaldo. We want to find the mathematical rule to get p, when you know t.

1st: Describe the pattern in the time column.

2nd: Describe the pattern in the position column.

3rd: Use those patterns to find the relationship that connects the time and the position in each row.

t (time in seconds)	Rule to get p	p (position in cm)
0	0 × 6 + 5	5
1	|	11
2	|	17
5	|	35
10	|	65
t	t × 6 + 5	p

Equation: $p = 6t + 5$

B. For every second that Reynaldo travels, how many centimeters does he move? How does this relate to your rule?

Reynaldo moves 6 cm per sec

C. Write the equation that will work for any row that could be in the table.

The equation for finding p when you know t is:

$p = \underline{6}\, t + 5$

D. Write the three steps from question 3C in your own words. Use the same steps for Geneva and Bommakanti to find equations for their motions.

Geneva

t (time in seconds)	Rule to get p	p (position in cm)
0	0 × 4.5 + 10	10
1		
2		
3		
4		
5		
10		
t	t × 4.5 + 10	p

3rd:

Equation: $p = 4.5t + 10$

Bommakanti

t (time in seconds)	Rule to get p	p (position in cm)
0	0 × 8	0
1		
2		
3		
4		
5		
10		
t	t × 8	p

1st:

2nd:

Equation: $p = 8t$

4. For A–D below, use math to describe the motion for one character only. Do the following:

    Decide on a start position and speed for your character.

    Fill in the equation that will go with it.

    Sketch the graph that will go with it and label your axes.

    Use the file, *Better4.smw*, to see if you were right. If you were not right, correct your equation.

A.
Start position:

Speed:

Equation:

B.
Start position:

Speed:

Equation:

C.
Start position:

Speed:

Equation:

D.
Start position:

Speed:

Equation:

< ANSWERS WILL VARY >

Lesson Plan Day 5

Activities

Wendella's Journey	Group work	50 minutes

Big Ideas

- Multi-segment graphs can represent characters moving at different speeds.
- Graphs tell a story. Stories can be represented in the form of graphs. In this activity, students will learn to write stories from graphs and make graphs for stories.
- "Flat" lines represent standing still.
- Lines "slanting downward" represent moving backward.

Notes

Introduction

- The cell phone game *Wendella's Journey*, which will be about a little dog traveling in a magic forest, slower or faster depending on where she is.
- Make sure students get the three modes of motion: forward slowly, forward faster, stopped

Question 1A (5 minutes)

- If needed, guide students as follows
- take each segment,
- figure out which mode of motion,
- then write a sentence about what happened.
- The story need not contain any numbers; encourage creativity

Question 2 (15 minutes)

- Students find speeds for each segment in a trip.
- Watch for students to develop the following ideas, and have them share with the whole class:
- For the second segment, the speed is 0 because she traveled for 2 minutes and didn't get anywhere (her position did not change)
- For the third segment, reading the coordinates at the endpoint will not work—must find distance and time by counting boxes or subtracting.

Question 3 (5 minutes)

- Have students predict the motion before running the file.
- Help students build connection between "slanting downward" and moving backward.

✱ NOTE: Speed is calculated for each direction traveled, but the connection to negative velocity comes in another activity.

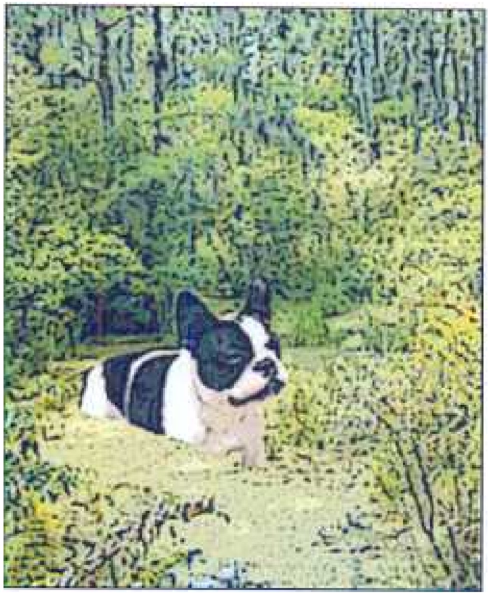
Questions 4, 5 (15 minutes)

- Students practice connecting stories and graphs and how moving backwards is represented.

EXTENSION: Encourage students to create more stories and graphs, using Wendella or different contexts if they can.

Wendella's Journey: Moving At Different Speeds

In our game, *Lost in the Pines,* Wendella the dog makes many journeys through the magical Lost Pines Woods.



On her journeys,

Wendella moves

– *Forward slowly* when she is in the swamp.

– *Forward faster* when she is on the road.

Wendella *stops* and barks for help when she is in quicksand.

We need many journeys and stories to use in our game. Help us set up the math and tell the story for each journey. Your work is very important. Remember—you are doing the math that will make the game work.

1. Open BUT DO NOT RUN the file for Wendella's first journey, *Wendella1.smw.*

   Using the graph for this journey, predict how Wendella will travel. Finish the story below.

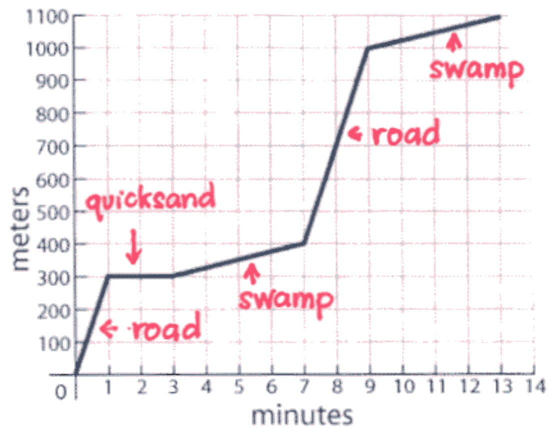
Wendella started out fast on the road. She was happy to be on her journey. Then...

<ANSWERS WILL VARY BUT SEQUENCE SHOULD BE:

road → swamp → quicksand → road >

2. Here is a different journey that Wendella took.

A. Mark on the graph below, show when Wendella was in the swamp, in the quicksand, and on the road. Mark the times on the minutes axis.



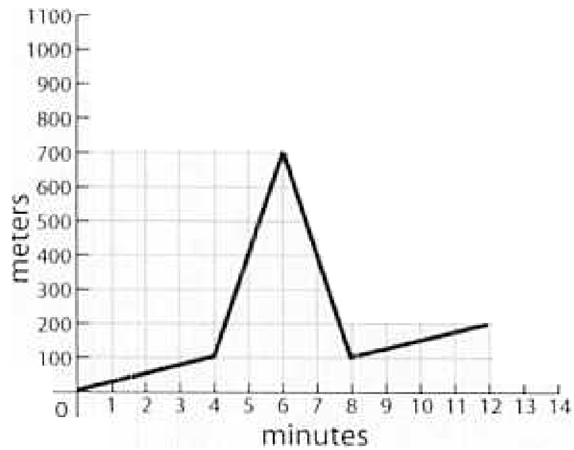
B. For each line segment in the graph above, find the number of minutes Wendella traveled, the number of meters she traveled, and her speed. You can make a table to keep it all organized.

SEGMENT	MINUTES	METERS	SPEED
1st	1	300	300 meters per minute
2nd	2	0	0 "
3rd	4	100	25 "
4th	2	600	300 "
5th	4	100	25 "

C. Choose one line segment in the graph—not the first one—and explain how you found Wendella's speed for that segment.

< ANSWERS WILL VARY >

3. Open the file *Wendella3.smw* for the third journey. The graph is also shown below.



A. Run the file. Wendella does something new here!
Write a story to go with Wendella's journey.

<STORY SHOULD INCLUDE BACKWARD MOTION
SEQUENCE SHOULD BE :

swamp → road → road backwards → swamp >

B. What did Wendella do 6 minutes after starting this journey?

Wendella went backwards

C. How does the graph show this motion?

Segment slants downwards

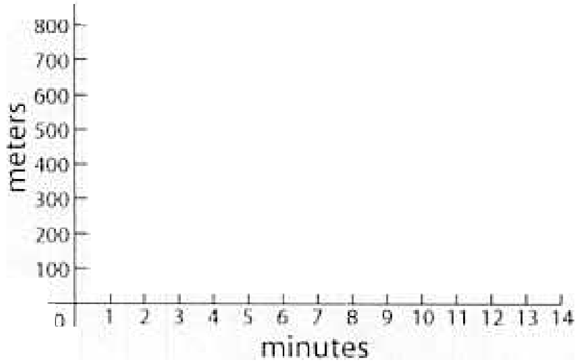
D. What is her speed between 6 and 8 minutes?

300 meters per minute

4. Open the file *Wendella4.smw*, and do the following.

   A. Change the graph so that Wendella goes forward and backward at least twice in her journey. Run the graph to make sure it works.

   B. Record your graph on the axes provided. < ANSWERS WILL VARY >



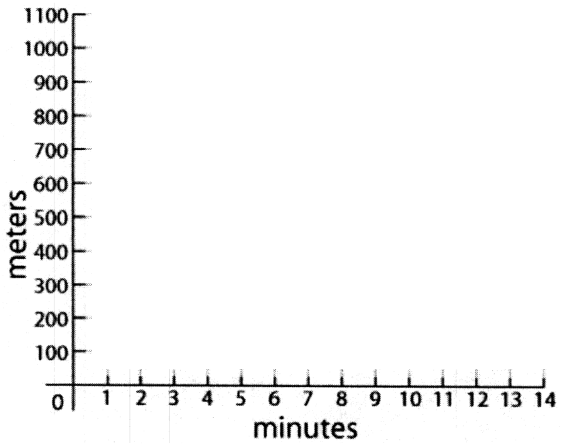
   C. Write a story to go with this Wendella journey.

< ANSWERS WILL VARY >

   D. One of the programmers doesn't understand what is happening in the graph. Explain how to use the graph to get Wendella to go backward in her journey.

Graphs should include a segment where the line slants downward

5. We need more journeys and stories. Using *Wendella5.smw*, make your own graph showing Wendella's next journey. Sketch the graph on the axes. Write a story to match your graph.



Story: < ANSWERS WILL VARY >

Lesson Plan Day 6

Activities

Money Matters	Group work	50 minutes

Big Idea

- Multi-segment graphs show varying speeds in motion contexts.
- Multi-segment graphs can also be used in non-motion contexts to show rate of accumulation, for example.

Graph	*Wendella's Journey*	*Money Matters*
x-axis	Time	Time
y-axis	Distance or position	Money accumulated
Steepness of line (slope)	Rate of motion: speed	Rate of accumulation: how much money went into (or was taken out of) the account that month

Notes

Introduction

- The new context here is the business of TexStar games.

✱ NOTE: We use line graphs to represent the changing balance of an account from month to month. The lines connecting the points for the monthly balance are basically meaningless. But it makes sense to draw the line to show the shape of the graph. This is a common practice among both mathematicians and math users such as accountants.

Question 1

- Make sure students understand that *balance* refers to the amount in the bank account.
- Help students interpret graph and table, highlighting the connection between them.
- Running the file will show the balance changing month by month, which can help students make sense of the graph.
- Keep asking: What is this part like in Wendella?
    - Increasing balance is like moving forward
    - Decreasing balance is like moving backward

Question 2

- Students apply lesson concepts to a new problem.
    - Make sure students understand that the graphs show how often the two games are played.
    - Question A is similar to parts of question 1.
    - For Question B: Ask for students' interpretation.
    - For Question C: Either game can be chosen, but the mathematical defense has to make sense.

Money Matters

We're glad you helped us with some great designs for our cell phone games. But TexStar Games has to make money too! Please help us with some business problems.

1. We keep a record of how much money we have in the bank. When we make money, we add it to our bank account. When we spend money, we take it out of our bank account.

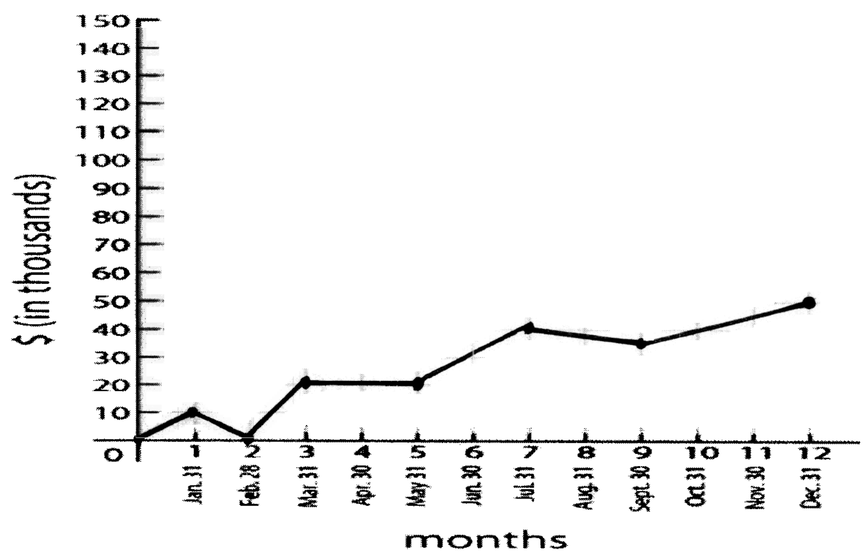
   The amount of money (in $) that we have in the bank on any day is called our bank account balance.

   On the last day of each month, we check our bank account balance and we graph it, comparing month to balance.

   Help us understand what happened to our bank account this year.

   A. Open the file *Bankaccount.smw*, which shows our bank account balance over the past year. Run the simulation to see how the bank account balance changed from month to month. Using the graph, simulation, and tables, answer the questions below.

      Fill in the graph below.



Find the following and explain how you know (from the graph, table, or simulation).

Beginning balance. $0	Ending balance. $50,000
Months in which our balance increased. January, March, June, July, October, November, December	Months in which our balance decreased. February, August, September
Month with highest increase in balance. March	Months during which the balance did not change at all. April, May
Month in which the balance was the highest. December	

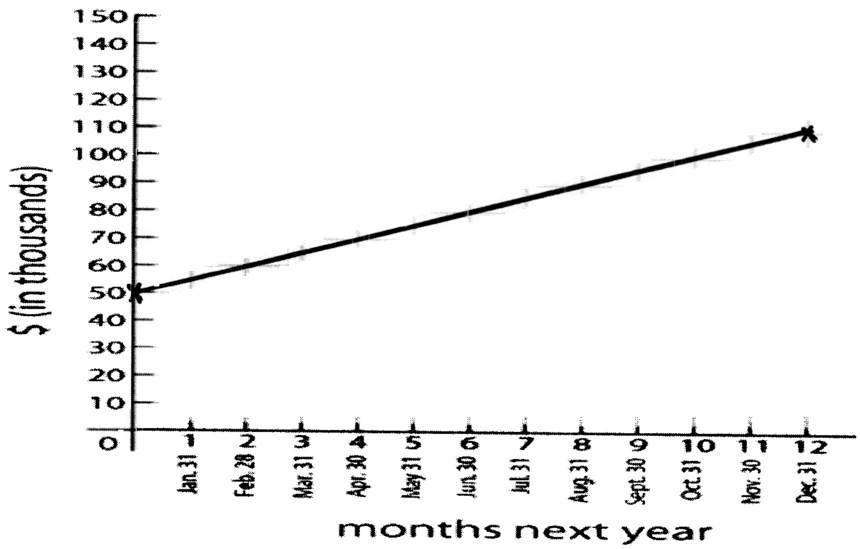
B. Write a sentence or two that describes what happened to our bank account balance over the year. Make up your own explanations for what might have happened. For example: The balance decreased between January and February. This is because we had to pay for a lot of artwork for a new game.

< ANSWERS WILL VARY >

C. Our goal was to have $60,000 in our bank account by the end of the year. By how much did we miss our goal?

$10,000

D. Our goal for next year is to end the year with $110,000 in our bank account. We would like to put the same amount of money in the bank each month. Complete the graph below to show this.



E. Use the graph you made to answer the following questions.

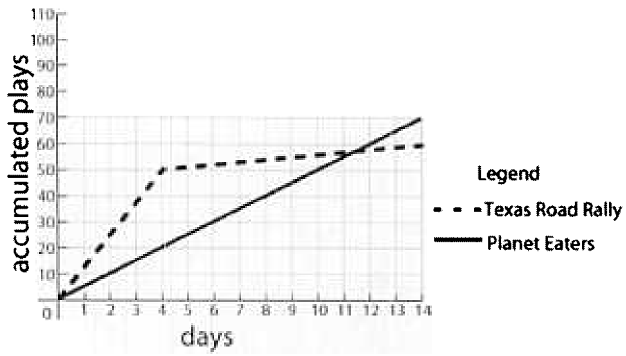
How much money do we need to put in the bank each month to reach our goal?

$5,000

How can we predict what our balance will be each month?

$y = 5,000x + 50,000$

2. We have two new games, *Texas Road Rally* and *Planet Eaters*. Next month, we want to start selling just one of them. To help us decide which one to sell, we let kids in the Electronic Gaming Afterschool Club use our games for 14 days. At the end of each day, we graphed the total number of times each game had been played altogether.



A. About how many times had each game been played after

	TEXAS ROAD RALLY (TRR)	PLANET EATERS (PE)
• 1 day?	12 or 13	5
4 days?	50	20
10 days?	55	50
14 days?	60	70

B. Compare the kids' use of the two games over the 14 days.

Kids used PE for the same number of times each day.
They used TRR a lot more each day for the first 4 days, but they slowed down afterwards.
Overall, they used PE most.

C. We care about selling games kids really like to play. Using the information in the graph, advise us: Which game should we start selling next month? Make your own decision, and defend it with mathematics. (There is no one right answer.) <SAMPLE ANSWERS>

① Choose PE because it was used more overall.

② TRR was more popular initially. Choose TRR if you need money fast.

Lesson Plan Day 7

Activities

Mathematically Speaking: Graphs to Know	Group work	20 minutes
Crab Velocity	Group work	30 minutes

Big Ideas

- Velocity is speed with direction.
- Negative rates indicate backward motion.
- Position can also be negative, with 0 indicating some defined point such as a start line or water level.

Notes

Graphs to know

- Students build rapid graph interpretation skills.
- Help students make the connection between speed in motion context and rate of accumulation in non-motion contexts.

✱ Remember: each row should have a similarly shaped graph.

EXTENSION: Ask students to create contexts/ stories that would yield graphs similar to those presented in the activity.

Crab Velocity

Introduction

The crab cell phone game will have crabs moving up and down out of the water, while players try to catch them. We are creating the graphs and equations to control the motion of the crabs.

Velocity is described in the box. The idea will be developed in the activity.

✱ Revisit equations from Controlling Characters with Equations, if needed.

Question 1

- Highlight for students that the absolute values (need not use the term) of speed and velocity are the same.

Question 2

- *Crab2.mw* shows five crabs: some are going up; some are going down; some have negative start positions.
- Guide students in placing negative start positions in equations. Have students count squares across the 0 line to get the rate of distance per unit of time—velocity— or m in y=mx+b.

EXTENSION: Have students write equations for the remaining 3 crabs.

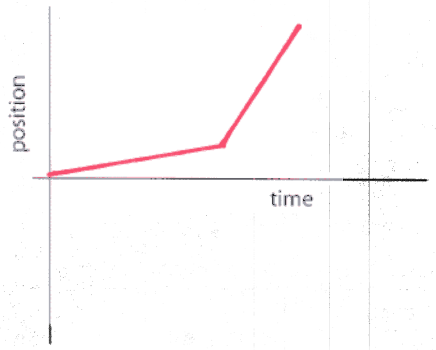
Question 3

Practice in class or homework.

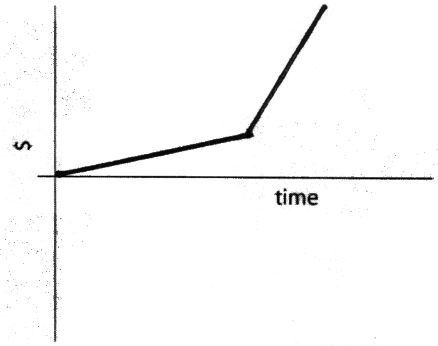
Mathematically Speaking: Graphs to know

In order to understand graphs, it helps to know some "by sight." Sketch a graph for each of the following movements. Use what you learned in the Wendella game and the Bank Account activity to think about these more general situations.

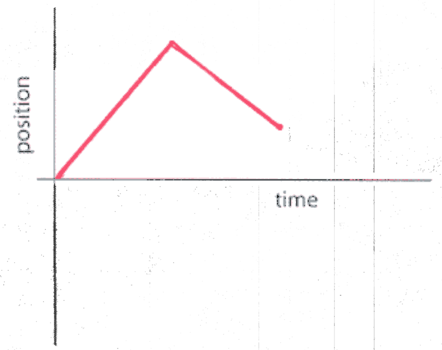
An object moving, then speeding up.



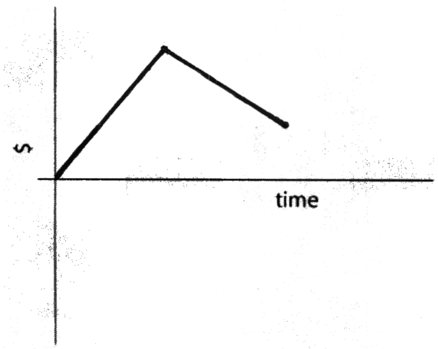
A bank account with its amount increasing and then increasing at a higher rate.



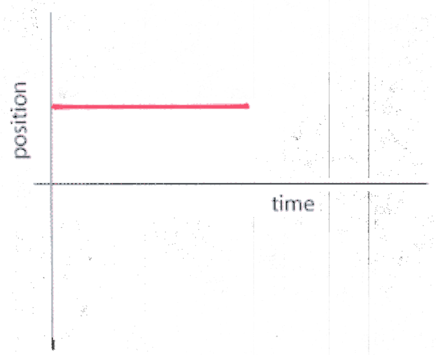
An object moving forward, then moving the backward.



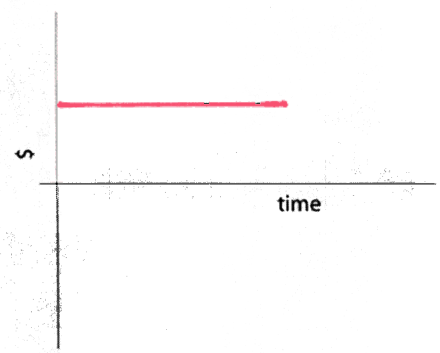
A bank account's money increasing and then decreasing.



An object standing still (not moving).



A bank account whose amount is not changing.



Crab Velocity

In our other games, characters can move forward and backward, or up and down. We already know how to calculate their speed. But there's a bit more to it than that. Help us learn how to use velocity in our games.

Velocity?

It's just speed—but with direction!

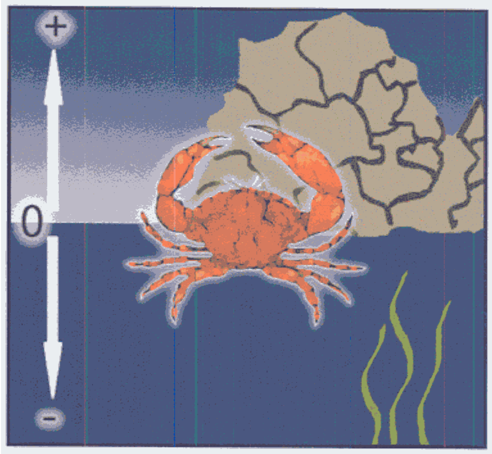
+ (positive) for up or forward

- (negative) for down or backward

Our crab characters, Karla and Lenny, move up and down—above the water (on the rocks) and below it. The water level is at 0 meters.

Positive positions are above the water line.

Negative positions are below the water line.



1. Open the file *Crab1.smw*, run the simulation, and find

Karla's speed: 0.75 m per sec or 3/4 m per sec

Lenny's speed: 0.75 m per sec or 3/4 m per sec

Karla's velocity: + 0.75 m per sec or + 3/4 m per sec

Lenny's velocity: – 0.75 m per sec or – 3/4 m per sec

2. Open file *Crab2.smw*. The graph shows the motion of Lenny, Karla and their children Dolores, Antonia, and Orlando.

   A. Find the velocity of each crab in the family.

   Karla: + 0.75 or + 3/4 m per sec

   Lenny: 0.75 or − 3/4 m per sec

   Dolores: − 0.40 or − 2/5 m per sec

   Antonia: + 0.40 or + 2/5 m per sec

   Orlando: + 0.40 or + 2/5 m per sec

   B. Using what you already learned, write the equation for Antonia's motion.

   $y = 0.4x$

   C. Predict the equation for Dolores' motion. Use the algebraic expression window to see if you are right.

   $y = -0.4x$

   D. Compare the equations for Antonia's and Dolores' motions.

   Antonia & Dolores move at the same speed

   BUT

   Antonia has a positive velocity & starts at (0,0)

   Dolores has a negative velocity & starts at (0, −5)

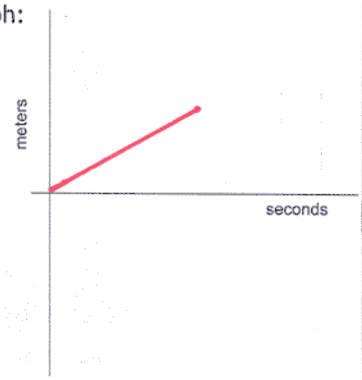
   or 5 meters below water level

3. Using *Crab3.smw*, make a graph for each situation below. Then, sketch the graph and write an equation for your graph. Remember, the water level is at position 0.

< ANSWERS WILL VARY >

Crab going up, starting from water level.

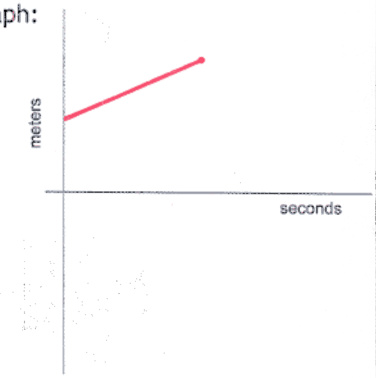
Graph:



Equation:

Crab starting from above water level and going up.

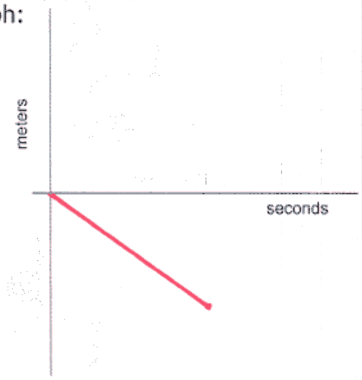
Graph:



Equation:

Crab going down, starting from water level.

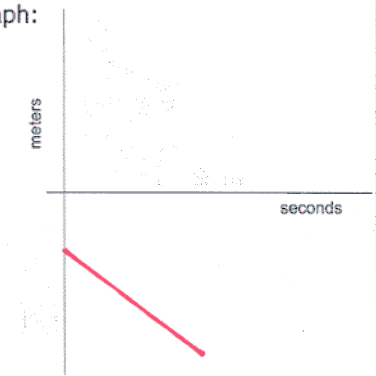
Graph:



Equation:

Crab starting from below water level and going further down.

Graph:



Equation:

4. Open *TemperatureGauge4.smw*. The graph shows the temperature inside TexStar Games' supercomputer used for running simulations. Zane, our supercomputer technician, can't run a simulation unless the computer's temperature is at least 30 degrees below freezing (-30 degrees Celsius).

   A. Find the speed of the temperature change in the supercomputer.

   5°C per minute

   B. Find the velocity of the temperature change in the supercomputer.

   5°C per minute

   C. If the temperature inside the computer starts at 0 degrees Celsius and the rate of temperature change stays the same, how long must Zane cool the computer before he can run a simulation? You can change the graph to help you, but you must keep the rate of temperature change the same.

   6 minutes

   Explain how you got your answer:

   The temperature must change 30°C

$$\frac{30\,^{\circ}\mathrm{C}}{5\,^{\circ}\mathrm{C}\text{ per minute}} = 6 \text{ minutes}$$

   D. Change the graph so that the starting temperature is 20 degrees Celsius. If the rate of temperature change stays the same, how long must Zane cool the computer before he can run a simulation?

   0 minutes

   Explain how you got your answer:

   <SAMPLE ANSWER>

$$\frac{50\,^{\circ}\mathrm{C}}{5\,^{\circ}\mathrm{C}\text{ per minute}} = 10 \text{ minutes}$$

Lesson Plan Day 8

Activities

Wolf and Red Riding Hood	Group work	50 minutes

Big Ideas

- Finding velocity of a character given some conditions.
- No matter how the characters move, if their motions graphs have the same endpoint, they meet at the same place at the same time.

Notes

Introduction

- Explain that in this activity, students will assume the role of the game tester.
  - We are figuring out in advance how to know what the right answer is.
  - Players won't have access to our mathematical representations, but they will make it easy for us.
  - Doing math enables players to make strategic decisions that will help them win.
- Read the introductory text that explains the rules of the game.
  - RRH and Wolf move across the screen to Grandma's House.
  - RRH's speed changes.
  - To win, players have to pick a speed for Wolf so that he arrives at Grandma's House at exactly the same time as RRH.

Question 1 (10 minutes)

- Ask students to give a story for the trip, with numeric estimations of RRH's velocities.
- Collect different predictions for the Wolf's speed, with a brief explanation of each prediction. Don't identify the right answer yet.
- Try different predictions out by setting the speed on the graph and verifying this with the simulation.
- Students may notice features of the graph that help: the two lines must intersect at the position of Grandma's house order to make a winning game.

Question 2 (10 minutes)

- Help students articulate and practice this idea:
  - When the endpoints of the graphs for RRH and Wolf are the same, that means Wolf and RRH are in the same place at the same time.

Question 3 (10 minutes)

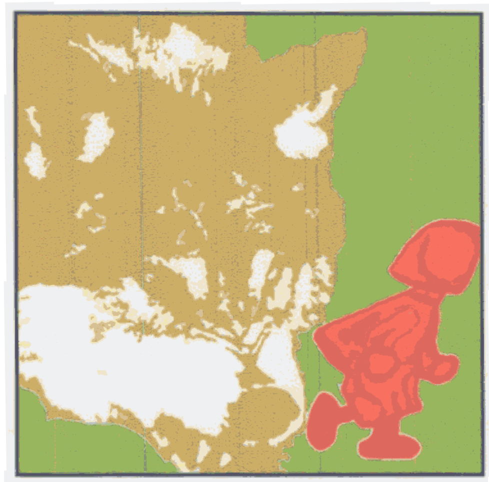
- Help students compare making a winning game from graphs or equations.
- Graphs are easier because you can draw the line and calculate the speed from the line.

✱ Pointing toward Algebra: this activity deals with simultaneous equations, very informally.

Question 4 (5 minutes)

- Help students develop these insights:
  - With graphs you can see where the Wolf's line begins and ends. Read the velocity off the graph once you have set it up.
  - With equations, predicting the Wolf's velocity is harder (with the methods we have now.

Wolf and Red Riding Hood



We need you to test a new game based on the fairy tale, Red Riding Hood.

In this game, Red Riding Hood (RRH) and the Wolf start from Home and move toward Grandma's House. RRH moves at two velocities during the journey. The Wolf moves at the same velocity the whole time.

The Wolf *must arrive at Grandma's House at exactly the same time as RRH.* Then he can devour her.

Help us find the "trick" so that the Wolf always arrives at exactly the same time as RRH.

1 Open *Wolf1.smw*. Use the graph to:

A. Describe RRH's trip to Grandma's. Include how long it took her to get there and estimates of her two velocities.

RRH traveled 600 m in 10 minutes
1st : 300 m in 4 mins so her velocity was + 75 m per minute
2nd : 300 m in 6 mins so her velocity was + 50 m per minute

B. Predict the Wolf's velocity so that he and RRH meet at Grandma's, at exactly the same time. <ANSWERS WILL VARY>

CORRECT PREDICTION : 60 m per minute

C. Test your prediction by changing the Wolf's graph and running the simulation. Were you right? Why or why not? Revise until you get it right.

<ANSWERS WILL VARY>

2. The next round of the game is *Wolf2.smw*. Again, there are two velocities for RRH; we need to find the one for the Wolf to catch her at Grandma's House.

   A. Use the graph to find the Wolf's velocity.
   What is the Wolf's velocity? 45 m per minute

   B. Explain how to use graphs to find the Wolf's velocity, when you have a graph of RRH's journey.

   The endpoint of Wolf should match the endpoint of RRH. Wolf's line should be a single segment.

3. Use *Wolf3.smw* to predict the Wolf's velocity with an equation.

   A. Using the algebraic expression window, predict the Wolf's velocity, check and revise until you get it right.
   What is the Wolf's velocity? 40 m per minute

   B. Explain how to use equations to find the Wolf's velocity, when you have an animation of RRH's journey.

   <SAMPLE ANSWER>
   Look at the total distance RRH traveled & divide that by the total time she traveled.

4. We can use graphs or equations to find the Wolf's velocity. Explain the advantages and disadvantages of each.

	EQUATIONS	GRAPHS
ADVANTAGES:	✓ you can input precise numbers ✓ you guess only 1 number - the number before x, where $y = (\text{number})\,x$	✓ once you know the endpoint, you just draw a single segment
DISADVANTAGES:	* guess & test	* you have to graph RRH's velocities so you know where Wolf's endpoint should be * the thickness of a line can make you think you are on a particular whole number but you might be off slightly

Lesson Plan Day 9

Activities

Secrets of Average Rate Revealed	Group work	50 minutes
Problem Solving	Group work / Homework	
Problems from the TexStar Lunchroom	Group work / Homework	

Big Ideas

- Average is the single number or rate that could "stand in for" the number or rates being averaged.
- If a character travels at several rates, the average of those rates gets the character to the same place at the same amount of time.
- A graph can be used to find the average rate (velocity) of a character moving at different rates (velocities): draw a line from beginning point to ending point of the characters' graph, then read that rate from the graph.

Notes

Secrets of Average Rate (25 minutes)

Question 1

- Remind students of the procedure for finding average that they have already learned in earlier grades: "add them up and divide" is fine here

Question 2

- Expect guess and check or other methods. Let students discover the connection between average and "one charge" through the activity
- For Question 2C: Students are likely to "add them up and divide" because the question asks for average.
- Connect average amount to single amount. These amounts can be thought of as rates, rates of dollars per game.

Question 3: Have students use the same graphs from Wolf and RRH to find RRH's average rate.

✱ The Wolf's single velocity IS RRH's average velocity. This insight makes it easy to find the average rate with the graph.

More Problem Solving (15 minutes)

- Question 1 provides practice for finding average rates from graphs.
- Question 2 uses average rate idea in a problem solving/modeling setting.
- Question 3 consists of problems that review earlier activities.
- For the rest: practice in class or give as homework.

Problems From The TexStar Lunchroom (10 minutes)

This can be used as homework or an informal assessment/quiz—or in class, if there is time.

- Question 1: Students use a unit rate to solve problems for particular values, and then to create a graph that generalizes. Help them see that the graph provides answers for any amount of coffee or cups.
- Question 2: Students practice translating from algebraic symbols to words.
- Question 3: Students practice filling in values in a table representing a proportional relationship. This can be surprisingly difficult for some students.
- For the rest: Again, students fill in values for a proportional relationship, but it's given in a somewhat unusual way and students must evaluate each time value to solve a problem.

Secrets of Average Rate Revealed

We need to use averages a lot in our work. You've already learned about averages in math class. And, you know something about them in real life. Use what you already know to find out more about average rate.

1. When playing a game *Robots Gone Awry*, Deborah gets scores of 70, 87, and 99. Use what you learned in math class to find her average, or mean score. Explain how you did it.

   70
   87
   99
   256 → 3)256.0 = 85.3 (−24, 16, −15, 10, −9) → Average is 85.3

2. Some game designers get paid for each game they make, charging different amounts for different games. In January, the designer, Sandro, sold 3 different games.

Name of the Game	Charge for the Game
Lost in the Pines	$10,000
Robots Gone Awry	$20,000
Texas Road Rally	$6,000

A. What was total amount of money Sandro was paid in January from selling these games?

$36,000

B. Sandro would like to charge the same amount for each game. What would he have to charge for each game, in order to make the same total amount of money for January?

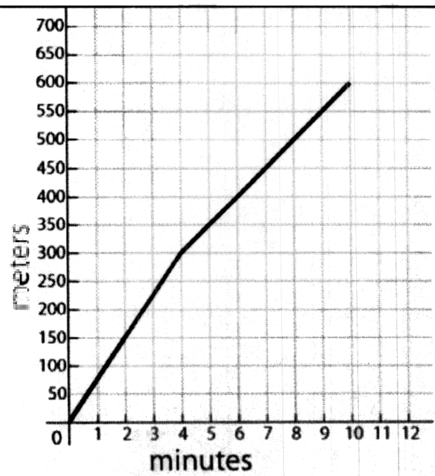
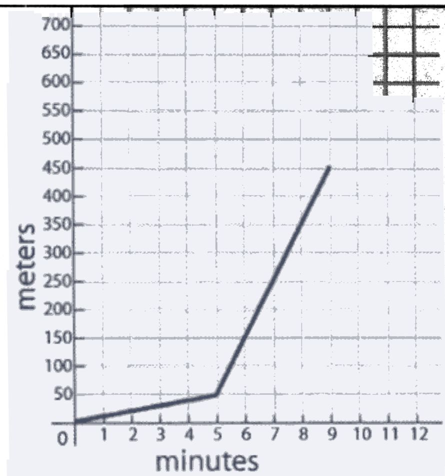
$12,000

C. What's the average rate (in dollars per game) that Sandro was paid, in January? Explain how you know, in two different ways.

$12,000 per game

3. Speed and velocity are rates, too. *Red Riding Hood and the Wolf* can help us think about average rate, using graphs. RRH's average rate is the single rate at which she could make the same journey in the same amount of time.

   A. You've seen these graphs before. Each graph shows RRH moving at two different rates.

	
First, RRH moves at 75 meters per minute for 4 minutes. Then, she moves at 50 meters per minute for 6 minutes. Predict: What is her average rate, in meters per minute? 60 m per min (Hint: What was the Wolf's rate, in meters per minute?)	First, RRH moves at 10 meters per minute for 5 minutes. Then, she moves at 100 meters per minute for 4 minutes. Predict: What is her average rate, in meters per minute? 50 m per min (Hint: What was the Wolf's rate, in meters per minute?)

   B. Look at the work you just did in Part A. In a sentence or two, explain the relationship between RRH's average rate and the Wolf's rate.

They're the same

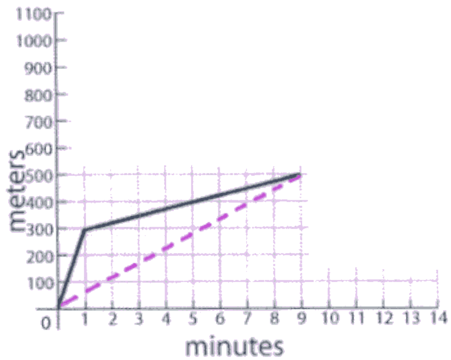
Problem Solving

We still have other problems to solve at TexStar Games! Use all you have learned to help us with the following problems.

1. Think back to *Lost in the Pines*. Help us analyze Wendella's journeys and average rate.

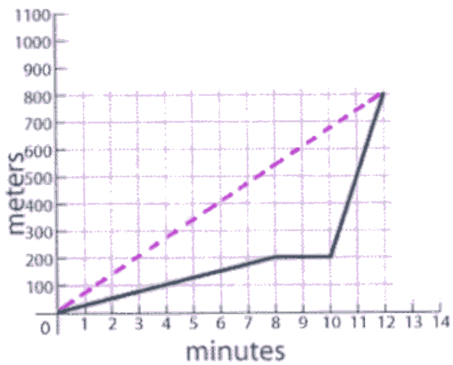
   A. What is Wendella's average rate of motion (her average speed) on this journey?

   about 56 m per min



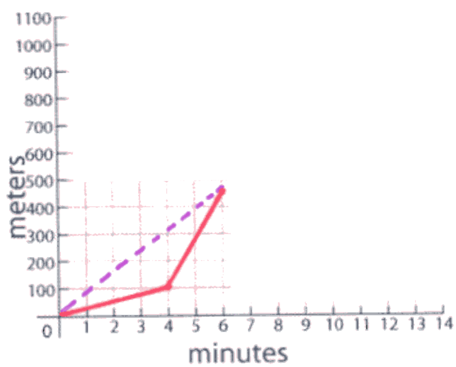
   B. What is Wendella's average rate of motion (her average speed) on this journey?

   about 67 m per min



   C. Wendella went 100 meters through the swamp in 4 minutes, then 350 meters on the road in 2 minutes. Using the graph to help you, what was her average rate, in meters per minute?

   75 m per min

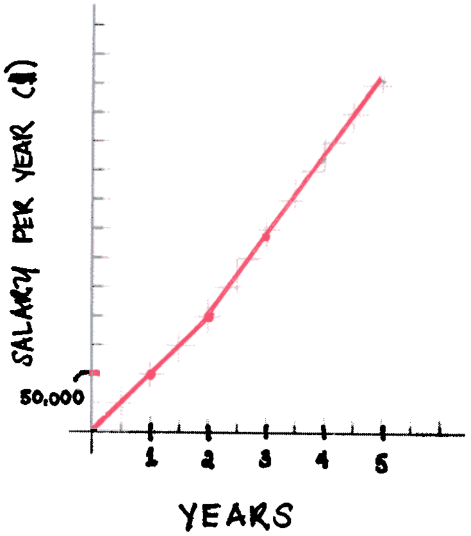


2 Marissa has worked for TexStar Games for 5 years. For the first 2 years, her salary was $50,000 per year. Then she got a big raise. For the last 3 years, her salary has been $70,000 per year.

Marissa says, "So my average yearly salary for these 5 years has been $60,000."

Her supervisor, Tanya, disagrees, "Actually, Marissa's average salary has been $62,000."

The $2,000 difference matters to them! Explain who is right and why (use the graph if it helps you).



The supervisor is correct.

$$\frac{\$\,310{,}000}{5 \text{ years}} = \$\,62{,}000 \text{ per year}$$

Problems from the TexStar Lunchroom

1 Our TexStar employees drink a lot of strong coffee while creating our great cell phone games. We know that with each 1.5-lb. (pound) bag of coffee, we can make 36 cups.

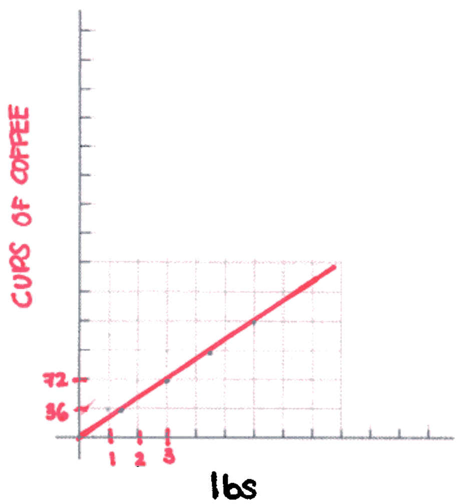
A. How many pounds do we need to make 72 cups? How about 120 cups?

3 lbs ↙ ↘ 5 lbs

B. If we have 3.5 bags of coffee left, how many cups can we make with it?

84 cups

C. Make a graph that lets us figure out how many pounds of coffee we need no matter how many cups we plan to make that day.



< ANSWERS WILL VARY BASED ON SCALE USED >

2. Shanae, the programmer, often writes in math symbols instead of words. She put up this ad on the company bulletin board:

Dog sitter wanted!
Will pay y dollars, where $y = 9x + 20$, and x is the number of hours you work.

Explain what Shanae means in words:

you get \$9 per hour plus a \$20 one-time bonus

3. Darrell, from the business office, is helping his son sell raffle tickets to raise money for his school. He's making a chart so that his son can quickly check that he has the right amount of money for the number of tickets he has sold. Fill in the missing numbers for him. There is no discount for buying a lot of tickets.

Tickets sold	Money Collected ($)
5	12.50
10	25.00
25	62.50
40	100.00

4. TexStar's company plane, the SuperNova, can fly at an average speed of 225 mph. It has only enough fuel for 3½ hours of flight.

A. Complete the table below.

City	Distance from Dallas	Travel Time	Non-stop Flight? (yes or no)
Cincinnati, OH	800	3.55 hours	no
Albuquerque, NM	580	2.58 hours	yes
Chattanooga, TN	675	3.00 hours	yes
Chicago, IL	795	3.53 hours	no
Athens, GA	780	3.47 hours	yes

B. Which cities can the SuperNova fly to non-stop (with only a single tank of fuel) if it starts in Dallas?

Albaquerque, NM
Chattanooga, TN
Athens, GA

Lesson Plan Day 10

Activities		
Mathematically Speaking— Linear Relationships: Proportional and Nonproportional	Group work/ Homework	50 minutes
TexStar Games: Going Full-Time	Individual Reflection	20 minutes

Big Idea

- Linear relationships can be expressed in the form y=mx+b and are represented as line graphs.
- In proportional linear relationships, b=0 and the line goes through the origin. Every pair (x,y) on the line is in the same ratio.
- In nonproportional linear relationships, b (does not equal) 0 and the line cross the y-axis at b. Though there is a constant rate of change in the equation, m, the pairs (x,y) are not in the same ratio.

Notes

Mathematically Speaking—Linear Relationships: Proportional and Nonproportional

Question 1 (5 minutes)

- Help students establish linear vs. nonlinear by sight for graphs and equations.
- Point out that students will study nonlinear equations more in later years. Here we use them for contrast with linear.

For Question 2 (10 minutes)

- Students uncover important distinctions between proportional and nonproportional linear functions.
- Help them summarize:
    - When the character starts at 0 and moves at constant rate, there is a proportional relationship between time and distance--any
    - When the character starts at a place other than zero and moves at a constant rate, there is a nonproportional relationship between time and distance.
    - In a proportional relationship, you can do one operation on any row (both x and y) to get another row. This does not work in a nonproportional relationship.

Question 3 (10 minutes)

Students organize the properties of linear relationships: proportional and nonproportional.

Question 4 (15 minutes)

Students classify some of the relationships they have used in the unit as proportional or nonproportional.

Question 5

Homework to give students practice labeling relationships as proportional or nonproportional, given just verbal description.

TexStar Games: Going Full-Time

- Give students an opportunity to reflect on the unit as a whole. Have them think about the math, but emphasize what *they* learned.

Mathematically Speaking
Linear Relationships:
Proportional and Nonproportional

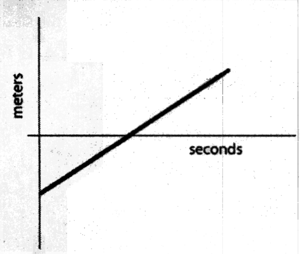
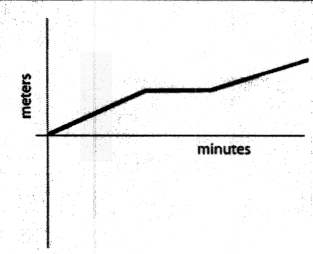
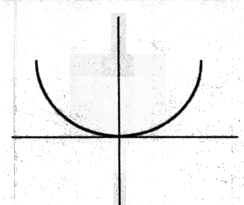
So far at TexStar Games, we have worked with relationships—for example, time and money in the bank, time and position of a character. It will help us to know mathematical terms for some of these different relationships.

Your task: Help us categorize relationships to make our work more efficient.

Linear Relationships vs. Nonlinear Relationships

1. In this unit, and in the math you will study in high school, we say:
   A linear relationship has a graph that is a single straight line.

Put a check by the representations of linear relationships and explain.

	
☑ Linear ☐ Not Linear Why?	☐ Linear ☑ Not Linear Why?
$y = 3x$	$y = x^3$
☑ Linear ☐ Not Linear Why?	☐ Linear ☑ Not Linear Why?
$y = 4x-3$	
☑ Linear ☐ Not Linear Why?	☐ Linear ☑ Not Linear Why?

Linear Relationships: Proportional vs. Nonproportional

2. Use this memo from Dora to help you think about linear relationships that are proportional and linear relationships that are NOT proportional. Reply to Dora in the spaces provided.

Hey - check this out. I have been looking at the numbers when a Planet Eater starts from the Sun (at zero) or some other place.

Table A shows the relationship between time and position when a character starts at 0.

Table A

Time	Position
1	3
2	6
4	12
5	15
10	30

x 2 (from row 1 to row 2, in both columns)

I worked out the equation as $y = 3x$ and I noticed a pattern between the rows here.

When time is 2, position is 6. Double 2 to get 4, and double 6 to get 12. Sure enough, when time is 4, position is 12.

So I can use doubling or multiplying to get one row from another. I think that works for any two rows in this table.

A. Try it yourself. Choose two rows and explain how to get one from the other using multiplication.

< ANSWERS WILL VARY >

B. Would this work with ANY two rows, even if I kept making new rows for this character's motion? yes

Why or why not? < ANSWERS WILL VARY >

Table B shows the relationship between time and position when a character starts at 2

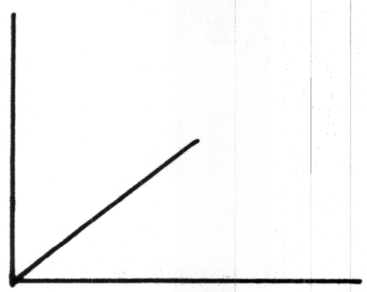
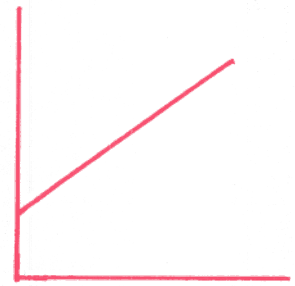
Table B

Time	Position
1	5
2	8
4	14
5	17
10	33

C. How about in this table: Can I double the numbers in a row, and get numbers from another row?

no

3. Dora uncovered two kinds of linear relationships. Fill in the properties of each below, using the relationship between time and distance in motions to help you think it through. <ANSWERS WILL VARY>

Motions starting at 0.	**Motions starting at some place other than 0.**
Example equations: $y = kx$, for any K	Example equations: $y = mx + b$
Example graphs: 	Example graphs: 
We call relationships with these properties **proportional linear relationships.** They have equations where: you can multiply any x value by a number to get a y value They have graphs that: start at (0, 0)	We call relationships with these properties **nonproportional linear relationships.** They have equations where: you multiply any x value by a number AND add another number to get a y value They have graphs that: do not start at (0, 0)

4. Go back through your workbook and find two **proportional linear relationships.** Write a sentence describing the relationship. Fill in the equation, table and graph for each. < ANSWERS WILL VARY >

x	y

Sentence:

Graph:

Equation:

x	y

Sentence:

Graph:

Equation:

5. Go back through your workbook and find **nonproportional linear relationships.** Write a sentence describing two of them below. Also, fill in the graph, equation and table for each.

< ANSWERS WILL VARY >

x	y

Sentence:

Graph:

Equation:

x	y

Sentence:

Graph:

Equation:

6. Label each relationship as proportional or nonproportional. You can use a graph, table, or equation to help you decide.

   A. A toy begins at 0 cm, the "start" line of a race, and moves at a constant rate of 3 cm per second.
   Relationship between time and the toy's position?

   proportional

   B. A character starts out 5 cm in front of the "start" line and moves at a constant rate of 3 cm per second.
   Relationship between time and the toy's position?

   non proportional

   C. A game designer earns $5,000 per month.
   Relationship between number of months on the job and total dollars earned?

   proportional

   D. A game design company has $50,000 in a bank account. They take $4,500 out of the bank to pay for each animation they need.
   Relationship between money in the bank account and the number of animations paid for?

   nonproportional

   E. You pay $10 to join the PlayCell game club. Then you pay only $1 for each game you buy.
   Relationship between total dollars ($) paid and the number of games bought?

   nonproportional

7. How did you decide whether a relationship was proportional or nonproportional?

   proportional relationships "start" at (0, 0)

TexStar Games: Going Full-Time

Your co-workers at TexStar Games were very pleased with what you have done. You've helped them improve their work, you are such a hard-worker, and you have also learned the math that is important to future success.

There are several openings at TexStar Games, and many people think you would be an asset to the company. Fill in the letter of application below so that Ms. Bahey, our Hiring Manager, will know your strengths and interests.

< ANSWERS WILL VARY >

Desired Position:

☐ Designer ☐ Programmer ☐ Artist ☐ Business Staff

Dear Ms. Bahey,

Thank you for the opportunity to work for your company. I have learned a lot about the challenges that come with a growing business like TexStar Games, Inc. I also realized how useful math can be in various aspects of your company. For example, *(list specific examples of what you learned)*

A number of TexStar employees have said that my math skills were valuable in addressing their problems. In particular, *(give 2-3 examples)*

In addition, I have other experiences and interests that can help improve your business. *(List examples like I am an avid cell-phone gamer myself; or I create computer graphics in my spare time.)*

I hope you will give my application your kind consideration. I am looking forward to helping TexStar Games create the most popular, fun and educational cell phone games in the country!

Respectfully yours,

____