Teacher Notes

Managing the Soccer Team:

A 7th-grade unit on rate and proportionality

Scaling up SimCalc

Managing the Soccer Team

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Contents

Introduction

Unit overview

In this unit, students develop proportional reasoning and understanding of rates by making connections among simulations of motion, graphs, tables and formulas. Students explore, dynamically, the relationships among these different representations by working with MathWorlds motion simulation software. The theme of managing a soccer team provides real-world contexts in which students can ground their reasoning.

The unit is organized into activities. Many of your lessons will include more than one activity. Some activities can be used as homework or independent work. The lesson plans we provide indicate how you can use the activities.

Materials

Teacher notes appear throughout the activities in a format similar to this: purple text on a gray background.

A Lesson Plan for each day appears before the student pages.

Daily student needs:
Rate workbook (provided)
Pencil and scratch paper
Graph and calculators
Straight edge

Computer use

Whole class discussions .... 1 computer with large display visible to all
Group work... 1 computer for every 2 to 3 students
Homework .... No computers needed

Classroom organization

Each activity is labeled with recommended use:

1. Whole class discussion, teacher led
   - Teacher and computer screen where everyone can see.
   - Everyone can hear everything everyone says. (This is the ideal!)
   - Students take notes in their workbooks.

2. Group work, teacher circulates
   - Students work in groups of 2 or 3, seated where each group member can see the shared computer screen and write in his or her own workbook.
   - Students should work together to come up with common solutions.
   - Each student should complete his/her own workbook.
     Group norms:
     - Take turns at computer keyboard, mouse.
     - Discuss your ideas. Keep discussing and asking until you understand the other students’ ideas. Aim for agreement based on reasons.
     - Your workbook shows your own thinking; it does not and should not be the same as everyone else’s.

3. Homework or independent work

- Students can work alone or in groups.
- Computers are not needed, so activity can be used at home or outside of classroom.
- Typically focused on practice of content in lesson, or generating data, or remembering old material relevant for the next day's lesson.

Many lessons begin with whole class discussion, then students use what they have learned in a group work activity. It is important to have students explain their answers in the whole class discussions and to encourage them to do so in their groups.

Mathematics goals

Students will:

- Understand rate in various situations as a proportional, multiplicative relationship.
- Identify rates in tables, graphs and equations relating two variables.
- Make connections among tables, graphs and equations.
- Use the various representations in solving problems, including comparing two phenomena with at least one having two rates over time.
- Understand and use conventions of graphing in interpreting and creating graphs.

Additionally, the materials will:

- Help students develop a view of proportionality as a function of two variables (y=kx).
- Develop students' problem-solving skills.

TEKS correlations

7.02.(D) Use division to find unit rates and ratios in proportional relationships such as speed, density, price, recipes, and student-teacher ratio.

7.02.(F) Select and use appropriate operations to solve problems and justify the selections.

7.02.(G) Determine the reasonableness of a solution to a problem.

7.03.(B) Estimate and find solutions to application problems involving proportional relationships such as similarity, scaling, unit costs, and related measurement units.

7.04.(B) Graph data to demonstrate relationships in familiar concepts such as conversions, perimeter, area, circumference, volume, and scaling.

7.05.(B) Formulate a possible problem situation when given a simple equation.

7.13.(A) Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.

7.13.(C) Select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.

7.13.(D) Select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

7.14.(A) Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.

7.14.(B) Evaluate the effectiveness of different representations to communicate ideas.

Timeline

Just suggestions, of course. Your mileage may vary!

Day 1	
Managing the Soccer Team	Whole class discussion
A Race Day	Whole class discussion
Another Race Day	Group work
Information Quest: How Fast?	Homework
Day 2	
Isabella Improves	Whole class discussion
Faster than Max	Group work
Practice Runs	Homework
Day 3	
Run, Jace, Run	Whole class discussion
Run, Jace, Run: Revisited	Group work/Homework
Day 4	
Back at the Office	Whole class discussion/Group work
Day 5	
Slope and Rate	Whole class discussion
Day 6	
On the Road	Whole class discussion
Road Trip Records	Group work
Day 7	
Graphs of Motion	Group work
Day 8	
Salary Negotiations	Whole class discussion
Summer Job Advice	Group work
All about MPG	Homework
Day 9	
How Far on How Much? MPG	Whole class discussion
Suiting Up	Group work
Day 10	
Manager’s Report	Group work
Mathematically Speaking	Whole class discussion

You can assign other homework as needed.

Lesson Plan Day 1

Activities

Managing the Soccer Team	Whole class discussion
A Race Day	Whole class discussion
Another Race Day	Group work
Information Quest: How Fast?	Homework

Big idea

Students connect a simulation of motion and its associated graph by understanding how time and distance are represented on the graph, and that each point on a line in the graph associates a time with a distance.

Notes

Managing the Soccer Team—whole class discussion

- Assure students they don't have to know anything about soccer.
- The mathematics they will learn will help them on the TAKS and in life: using rates, proportions and graphs are all important. Understanding connections among tables, graphs and symbols will help them in later mathematics they study.

A Race Day—whole class discussion

- Students takes notes in their workbooks as you lead the discussion.
- Introduce practice runs as the task.
- Introduce MathWorlds.
- Open araceday1.mw
  - Point out the simulation screen and graph screen.
  - Run simulation, noting how you do that.
  - Run again, using the step button, noting how you do that.
- *Ask: how is the simulation the same as and different from reality*?
  - Same in that a girl runs a distance in a certain amount of time.
  - Different: she stops at finish line exactly, no running past. She runs same speed the whole time.
- *Ask students to read the questions 1a-g*
  - Discuss each. See notes on student page. Answers given are those to aim for, but students may have different insights and ideas.

Another Race Day—group work

- Students work in groups at the computer. Before they start, hand out MathWorlds Quick Reference Guides. Point out relevant parts: open file, controls.
- Methods for learning to use MathWorlds:
  - Read the Quick Reference Card.
  - Play around with software.
  - Train a trainer. You show one student how to do something; the student quickly demos to each group.
  - Cascade training. You show one group: that group divides up, each teaching another group, until all have learned the skill.
- See notes on student pages.

Assign homework, Information Quest. Student should answer at least 2 items.

Whole class discussion

Managing the Soccer Team

From: Alamo Middle School principal

To:___________________________________ (you!)

Have we got a job for you! The soccer team manager just moved to Florida. The season is just about to start. Will you please be our acting soccer team manager?

There is so much to do:

- The players have to keep practicing for the games.
- We need help making sure the transportation goes smoothly and we are spending way too much money on fuel for the van and bus.
- Uniforms and other supplies are needed.

If you do a good job, we will offer you the job on a regular basis.

Are you up for the job?

For the next two weeks, **you** are the soccer team manager. In this mathematics unit on rate and proportionality, you will solve problems that are very close to what a team manager would actually have to do. (Of course we had to rig them up a bit to get all of the mathematics in.) You will use software called MathWorlds that helps you understand rates and proportionality using simulations, graphs and tables. And you will be learning mathematics important in the TEKS.

Good luck, have fun, work hard!

Whole class discussion

A Race Day

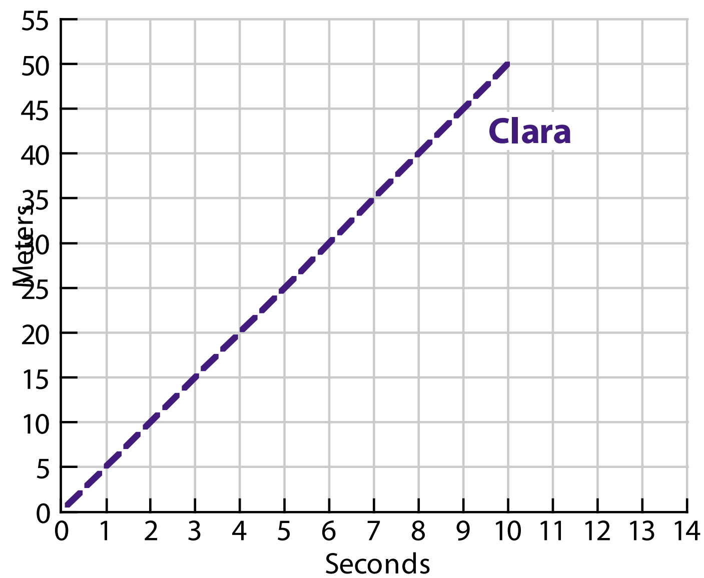
To improve their speed, the soccer team spent a few practices running dashes at the track.

MathWorlds can simulate some of these dashes and make graphs of them.

Carefully watch the simulations and graphs to answer the questions.

1. First up was Clara. Watch the simulation of Clara's dash.

Open and run the file, araceday1.smw.



a. What does the simulation show?

> **It shows a girl running. (This is the essential piece to get from 1a. More or less complex answers are possible) Also accept these:**
> - **It takes her 10 seconds to run 50 meters.**
> - **She runs at the same speed the whole time.**

b. What happens to the graph as the simulation runs?

> **The graph builds up from left to right. The line gets longer. The graph builds a line slanting up. (No need for students to use the word slope, yet. Just collect their informal ways of describing the line.)**

What do you think?

What is speed, anyway?

> **Speed is a number that describes how fast something is going. Speed is a rate comparing distance to time.**

c. How far did Clara run?

50 meters

d. Where do we find that distance in the graph?

On the y-axis

e. What does the x-axis show us?

The time in seconds.

f. How long did it take Clara to complete her dash?

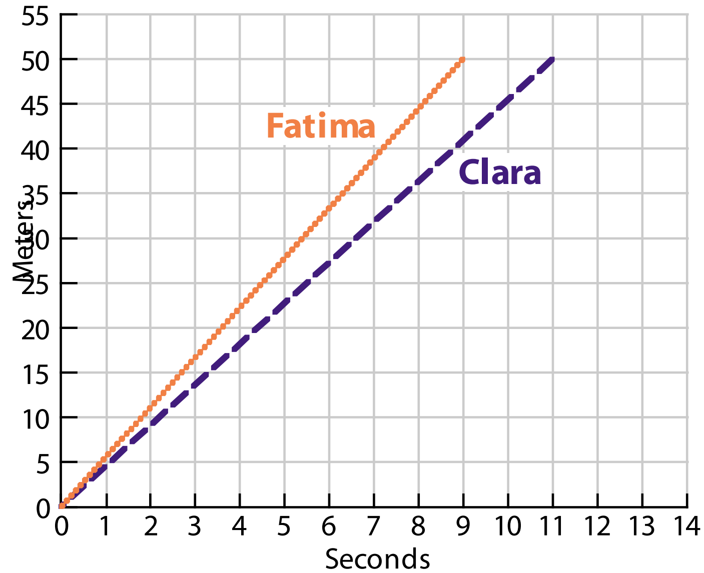
10 seconds

g. How can we figure out her speed?

Divide 50 meters by 10 seconds to get 5 meters per second.
Just recalling how to calculate speed is enough here. Show units on board. This will be the foundation for building understanding of rates and slopes later.

2. Next, Clara and Fatima ran a race.

Open the file, araceday2.smw, and watch the simulation of their race.



a. What does this simulation show?

Two girls running.

b. Who was going faster? Who won the race?

Fatima

c. What is the total distance of the race?

50 meters

d. How long did each girl take to complete the race?

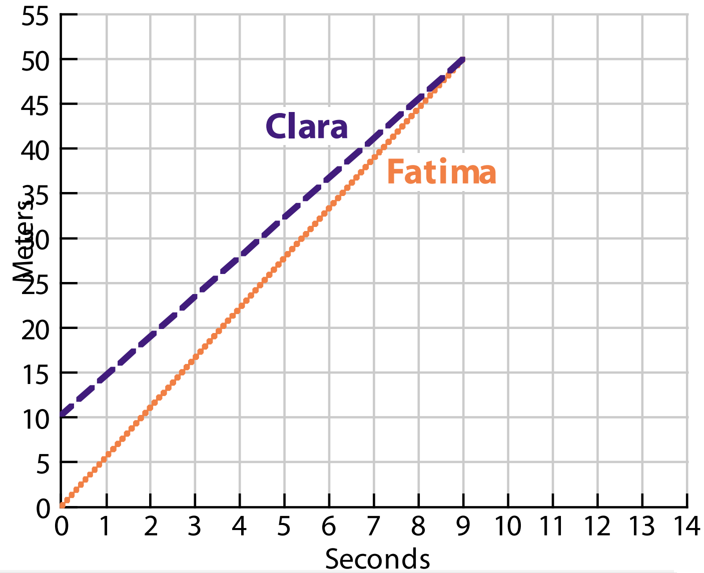
Clara—11 seconds
Fatima—9 seconds

e. Find the speed of each girl.

Clara 50/11= 4.55 meters/second
Fatima 50/9=5.56 meters/second (accept rounding decimals or fractions)

3. Clara and Fatima raced again.

Open the file, Araceday3.smw, and watch the simulation of their second race.



a. What happened in this race?

Clara got a head start, and they tied.

b. Who is going faster? How do you know?

Fatima: she had to run farther in the same amount of time. If students note that the line is steeper, or need to calculate, that's fine. The slope insight comes in later lessons.

c. Find the speed of each girl.

Clara: 40/9 = 4.44 meters per second
Fatima: 50/9 = 5.56 meters per second
Help students notice that they must count grid lines or subtract start distance from end distance to find Clara's distance.

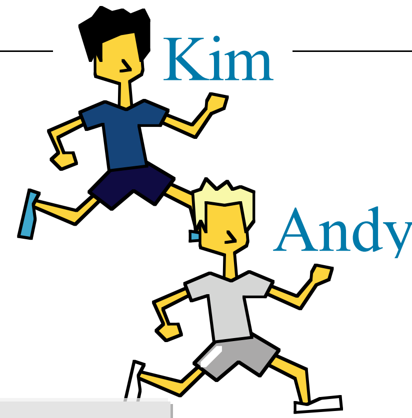
4. Compare the dash in 1 and the two races in 2 and 3.
What is the same? What is different?

This open-ended question can have many answers. Have students give responses and explanations, note if they are mathematically relevant, and make sure students compare graphs. You can categorize answers in terms of the graph and the race, and match them up.

Race simulation	Graph
◆ One vs. two racers.	◆ One vs. two lines.
◆ Same start vs. different start.	◆ Line "starts" at (0,0) vs. up the y-axis.
◆ Different speeds but all 50-meter dashes.	◆ Different slants, different "end" on x-axis, but all have endpoint (x, 50).
◆ Clara wins vs. tie race.	

Group work

Another Race Day



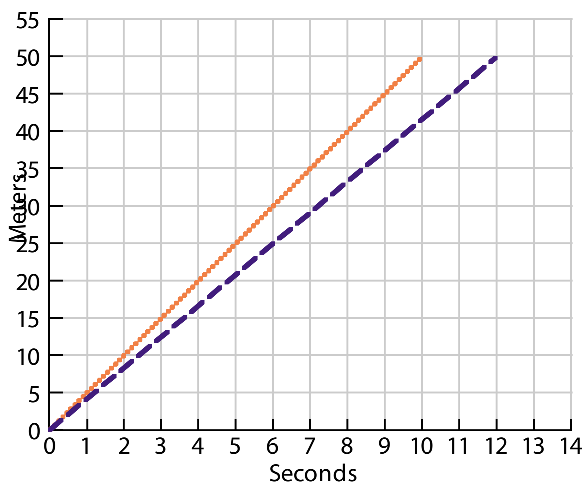
Andy and Kim are the next two players to try the dash.

Open and run the file, another1.smw.

1. Watch the race several times. Use the  step button. Describe what this simulation shows.

Any of the following should be encouraged: Two boys racing. Kim, with blue dot, wins. Kim steadily pulls ahead. Step shows that Kim runs approximately 5 meters each second. Andy goes a bit less than 5 meters a second. When Kim crosses the finish line, Andy is at about 42 meters (so Kim wins by about 8 meters or exactly 2 seconds). Andy crosses at 12 seconds.

2. Predict the graph: on the axes below, sketch a graph of Andy and Kim's dash.



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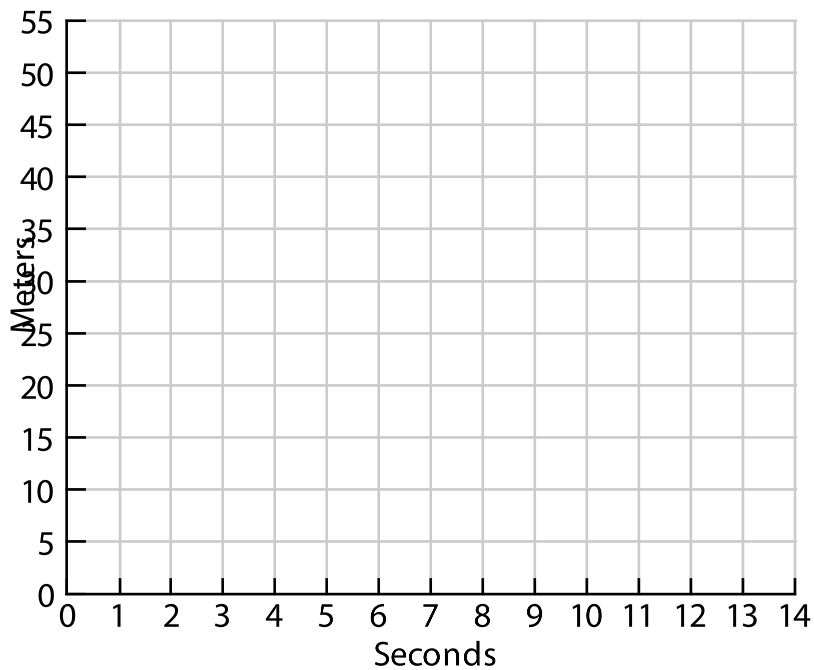
3. Open the file, another3.smw. This shows the runners as dots and gives you more information.

a. Use the step button and other controls to make a graph below that is more precise than your sketch.

> **Encourage students to make a table as they do this.**



> **There is a sample answer on the previous page. Kim's line (orange) should go through (1 second, 5 meters). Three points should be plotted to ensure a line. Andy's line (purple) should go through (6 seconds, 25 meters).**

b. Label three points in the graph above. List them below and explain what they mean in terms of the race.

> **Can vary but should have interpretation in terms of race itself.**

Point	In the race
(0,0)	**They start at the same place at the same time.**
(10,50)	**Kim ran 50 meters in 10 seconds.**
(6,25)	**Andy ran 25 meters in 6 seconds.**

Homework

Information Quest: How Fast?

Find out about as many of the speeds below as you can.

Try these resources: Web search, Encyclopedia, Almanac, County Extension agent, a stopwatch.

The purpose is to provide common information about typical speeds that students can use to gauge the reasonableness of their answers in later work. Precise speeds are not that important.

1. World's record, 100-meter dash

This will commonly be given in seconds; for example 11.1 seconds. Because the distance is 100 meters, then the speed is 100/11.1 meters per second. Students may come up with different answers depending on references cited; that's ok.

2. Average speed of a black ant

3. Record for high schooler's 100-meter dash

4. Typical walking speed, human

5. Speed limit on the road or street nearest your home

6. Your walking speed

Students will have to time themselves as they walk a certain distance. Ask for the distances and how they chose them.

7.

Make up some of your own.

8.

Lesson Plan Day 2

Activities

Isabella Improves	Whole class discussion
Faster than Max	Group work
Practice Runs	Homework

Big idea

For equal length dashes, as the time goes down, the speed goes up. Faster dashes are represented by steeper lines (lines with greater slope). These connections are built through calculation and understanding of the graph.

Notes

Isabella Improves—whole class discussion

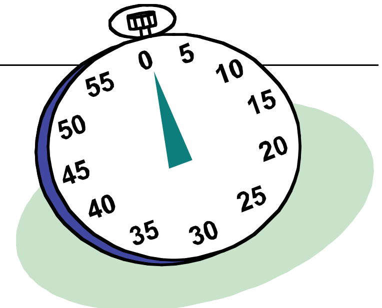
- This simulation shows all 4 dashes in the same "world" and the lines on the same graph. It also uses "dot world." Make sure students understand the representation.
- See notes on student pages for specifics.
- *End the whole class discussion by demonstrating how to change the steepness of a line* using the grey dots on the graph that control each runner's line.
  - Dot on graph line   Change slope
  - Dot on x-axis   Change length

Faster than Max—group work

- Students use the steeper/faster connection to construct a graph line for a runner, then use MathWorlds to simulate the dash.
- MathWorlds skills: Students need to use the grey dots that control the length and steepness of a line to adjust the "nub" that is placed for Nola in the file, fastermax.mw.
- Students then "work backwards" to calculate speed from the line. Each group or individual may have a distinct solution. This provides opportunity for students to try to understand someone else's work.

Whole class discussion

Isabella Improves

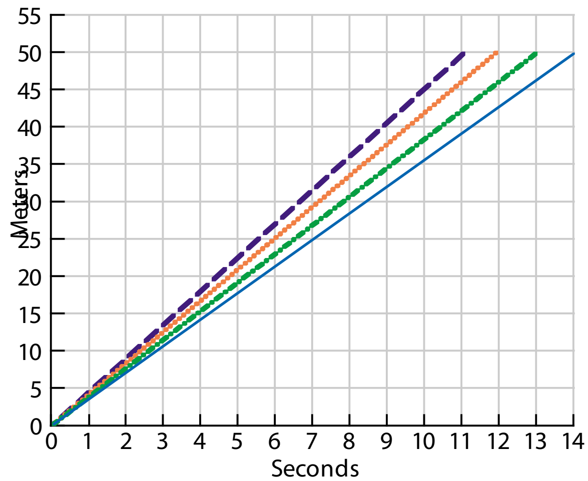


Isabella decided to run 50-meter dashes all week, trying to improve her speed.

Her first dash took 14 seconds. For the next three days, she improved her time by 1 second each day.

1. Predict the graph: On the axes below, sketch a graph that shows all four of her dashes.

Give students time to fill in this graph before you go on. Help them coordinate the axes to produce the pattern.



2. Open the file isabella2.smw. Run the simulation several times. Observe the relationship between the runners and the graph.

Note different wrong answers and the misconceptions from which they arise.

a. Was your prediction right? If not, explain why not.

As you run it, ask students to watch the simulation first, say what is going on, then focus on one Isabella run and its graph, then the pattern of all 4.

b. Describe what this graph and simulation show.

The simulation shows all Isabella's dashes at the same time. Each is 50 meters, but each has a different speed.
The graph shows shorter and shorter times for the same distance.

c. Describe patterns you see in the graph.

All lines start at (0,0). Shorter times result in steeper lines. The endpoint of each line is at 50 meters on the y-axis, but at different (1 second apart) places on the x-axis.

3. As Isabella's time went down by one second on each dash, what happened to her speed? Let's find out.

a. Fill in this table.

The speeds below were rounded to the nearest 100th. Students can do just to nearest 10th and still see differences.

Dash	Distance (meters)	Time (seconds)	Speed (meters/second)
1st	50	14	3.57
2nd	50	13	3.85
3rd	50	12	4.17
4th	50	11	4.55

b. Describe and explain patterns that you see in the table.

What to do
Describe: Say what you saw.
Explain: Say why.

As the time goes down, the speed goes up. This is because the distance, 50 meters, stays the same but is divided by a smaller number in each row, giving a larger quotient.
So as time goes down, speed goes up, when distance stays constant. Students may try to uncover number patterns, but this qualitative statement is the main point to take away. Point out the connection between this table and the graph.

4. Frankie, who manages the track team, heard about your dash graphing and wants to do it too. Give advice on how to do this.

> **The main point here: confirm graphing skills and speed calculation.**
> **Re-emphasize the steeper/faster connection.**

a. Describe how to graph dashes on time and distance axes.

> **If the runners start at the same place, then each dash line starts at (0,0). Find the ending time and distance on each axis, find the point representing these. Draw a line from (0,0)—or other start place—to the ending (time, distance) point.**

b. Describe how to use the graph to say which runner went faster. Explain why this works.

> **The fastest runner's dash is represented by the line that is the steepest. This is because less time is needed for each distance, which "lifts" the graph up.**
> **The explanations can be linked to the graphs intuitively, as in this answer.**

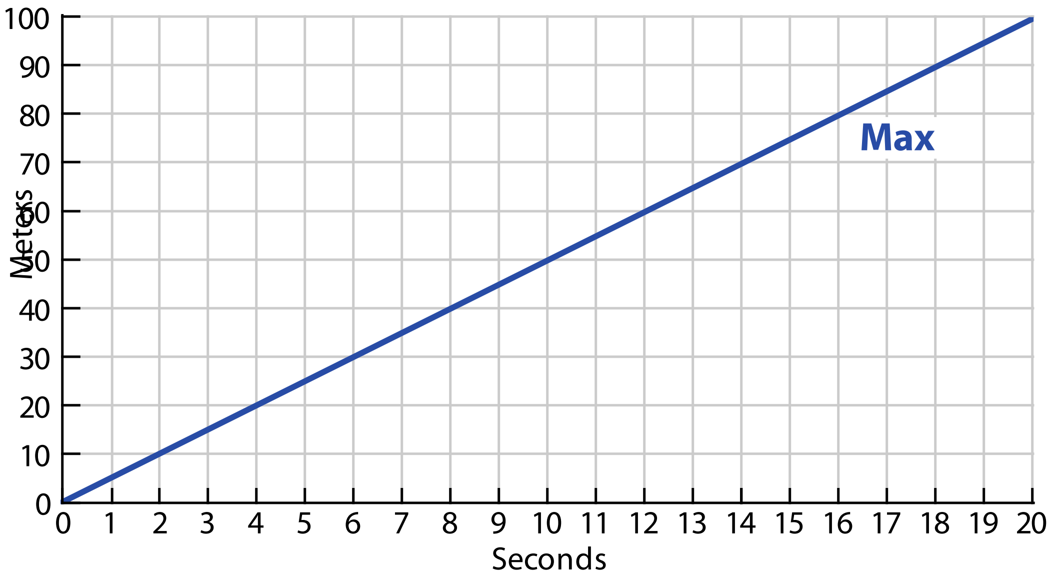
c. Describe how to calculate the speed of a runner.

> **You can stop if you get this: Speed is the distance divided by the time. Students do not need at this point to make the connection between slope and speed, or how to calculate slope.**

Group work

Faster than Max

Max and Nola are next to run dashes, but they run 100 meters. You keep time and record the following graph for Max.



1. All you know about Nola's time is that she won the race.
Open the file fastermax1.smw. The blue line shows Max's 100-meter dash.
The magenta line is for you to model Nola's race.

a. Make a line that shows that Nola won the race.

Use the grey dots on the x-axis and the end point of the magenta line. Some students may need individual help in changing the graph. Train a student who can train others.

b. Now run the simulation. How far ahead of Max was Nola when she crossed the finish line?

This question asks students to use the graph in a new way. One way to get the answer: Draw a vertical line from the endpoint of Nola's dash line to the x-axis. Find the y-coordinate of the point on Max's line that has the same x-coordinate as the endpoint of Nola's. This helps students understand that each point on the graph represents a time and place for the runner. Stepping through the graph can also give the answer, if the steps are the "right" size.

c. Draw the line representing Nola's run in the graph above, making sure to accurately plot 3 points.

d. According to the line you drew, how much faster was Nola than Max? Describe how you found out. For your simulation and graph, how much faster was Nola than Max? Describe how you found out.

2. Exchange workbooks with another group or student. Check their work on page 12. Answer the following:

a. Does the work for 1d match the graph drawn for 1c? Explain.

Example answer: Their graph had an endpoint at 10 seconds, 100 meters. At 10 seconds, Max was at 50 meters. So they are right.

b. Did the other student use different ways than you did to find the answer? Describe the difference in your methods.

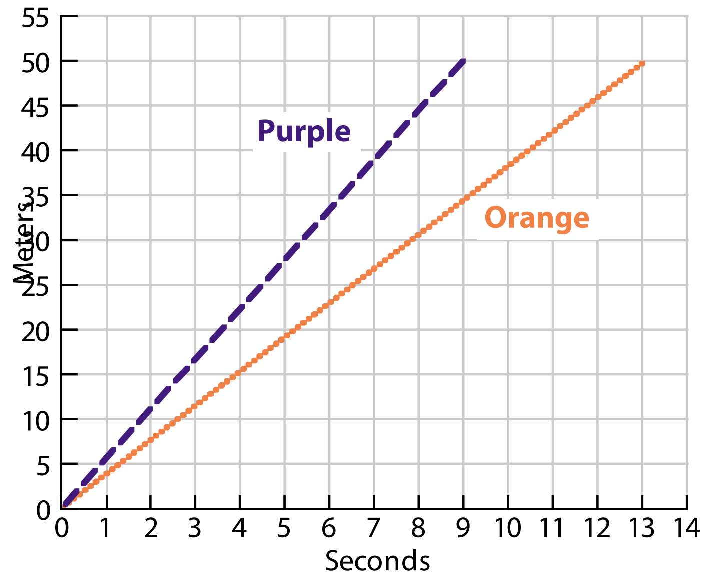
Homework

Practice Runs

1. The following graphs show two racers, Orange Runner and Purple Runner. Say who is going faster and who won for each race. Explain your reasoning.

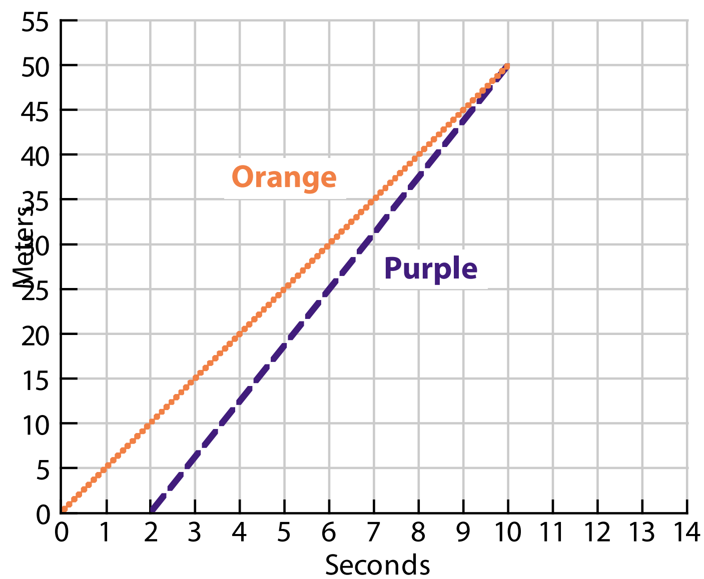
a.

Purple is faster.
Purple won.



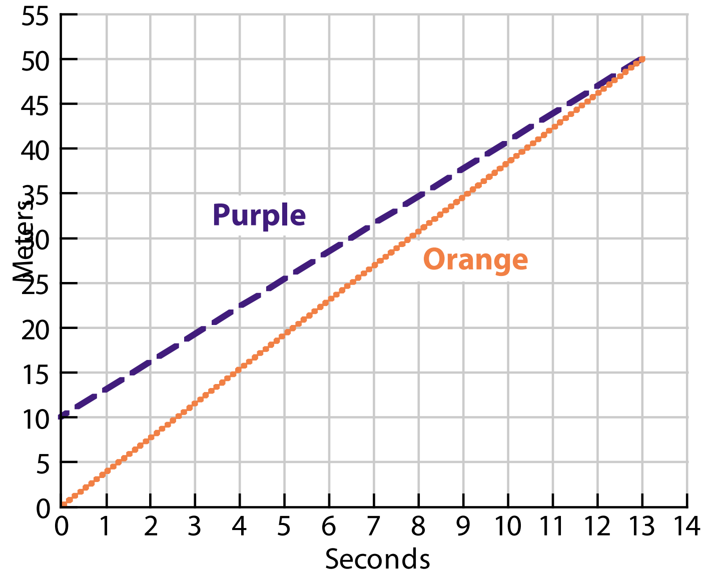
b.

Purple is faster.
They tied.
(Orange got a head start, purple started 2 seconds after Orange started).



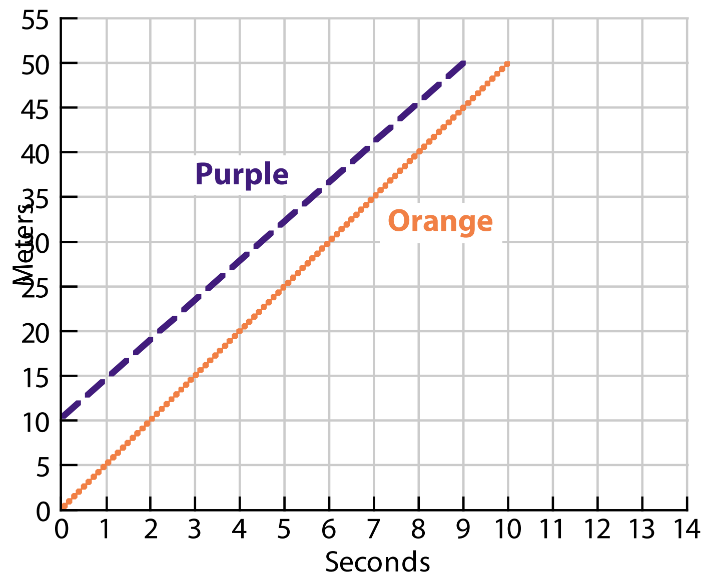
c.

Orange is faster.
They tied.
(Purple got a head start of a different kind. Purple got to start at the 10 meter mark, instead of at the starting line).



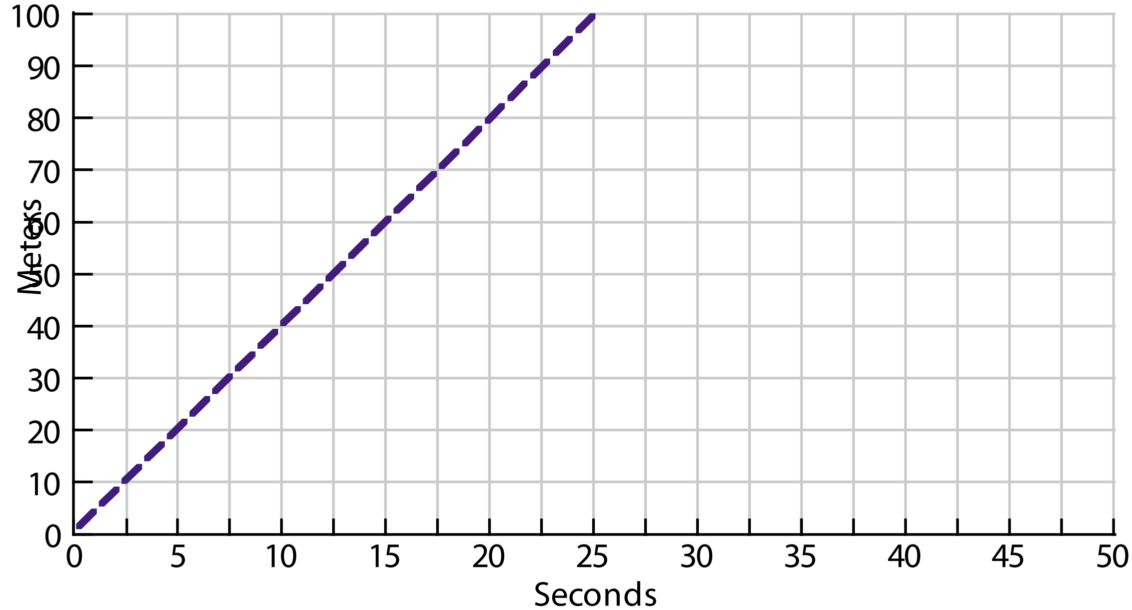
d.

Orange is faster.
Purple won.

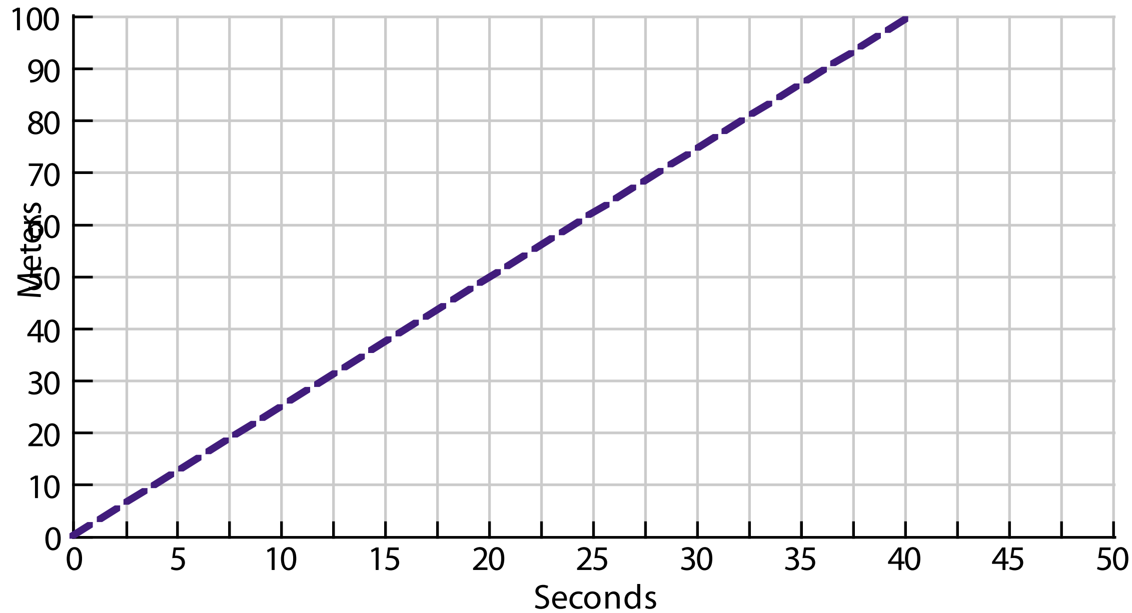


2. For each of the following descriptions, sketch a graph that would match the description.

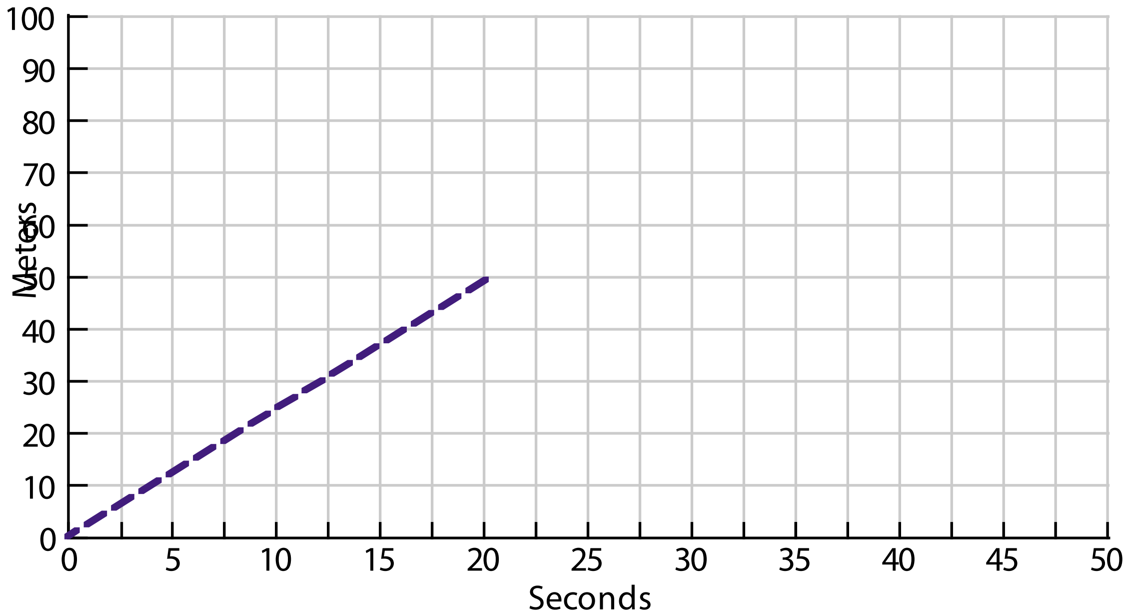
a. Ran 100 meters in 25 seconds



b. Ran 200 meters in 40 seconds



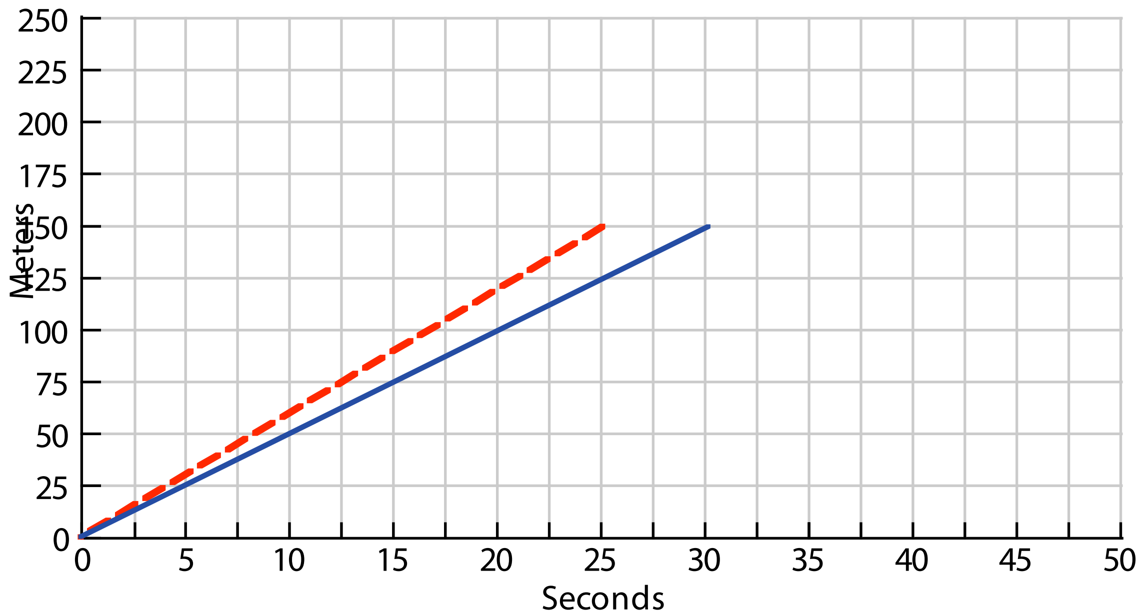
c. Ran 50 meters in 20 seconds



3. Draw graphs for the situations described in the following two problems.

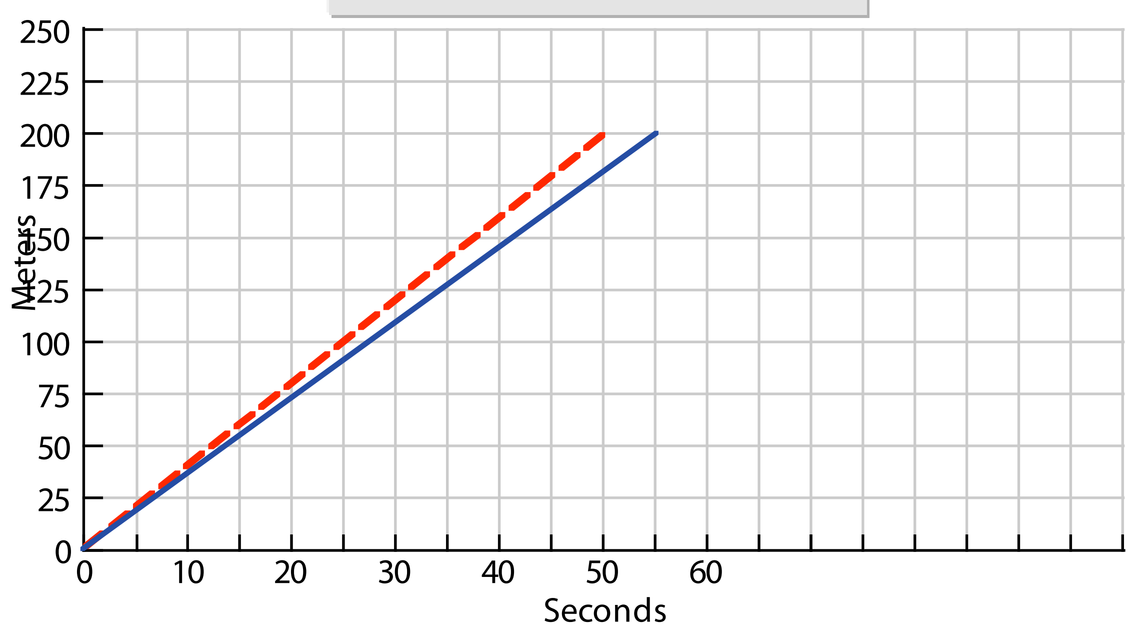
a. Mila and Shawntee ran 150 meters. Mila's time was 30 seconds. Shawntee's time was 25 seconds.

(Shawntee in Red, Mila in Blue)



b. Kim and Andy ran 200 meters. Kim did it in 50 seconds. She came in ahead of Andy by 5 seconds.

(Kim in Red, Andy in Blue)



Lesson Plan Day 3

Activities

Run, Jace, Run	Whole class discussion
Run, Jace, Run: Revisited	Group work/Homework

Big idea

Students explore proportional relationships in a table of times and distances. Moving beyond just calculating the speed by dividing final position by final time, they associate the speed with each row in the table, and express this with a formula for the first time. They get a chance to practice this new connection.

Notes

Run, Jace, Run—whole class discussion

- Begin the lesson by showing the simulation and graph of Jace's run. By now, students should be able to say quite a bit about the run, the graph, the speed, and the steepness of the line. Give them time to state all of these.
- Put the table from the workbook on the board; go through the questions, eliciting table entries as you go. Elicit alternate strategies as much as possible. Point out how the speed enables you to fill in any table row.
- Guide students from graph, to table, to formula. We use the term "formula" to describe the symbols that represent a function, as opposed to table or graph.
- Use 5, with its 3 tables, to help students get ready for "Run, Jace, Run: Revisited."

Run, Jace, Run: Revisited—group work/homework

- This matching activity reinforces the table, formula, graph connection.

Whole class discussion

Run, Jace, Run

1. Open the file runjace1.smw. Watch the simulation and graph of Jace's 100-meter dash.

a. Use the graph to answer: How many seconds has Jace run when he has gone 25 meters? How many seconds has he run when he has gone 50 meters?

The points here are easy to read from the graph or by using step and numerical display.

b. Now fill in the first two rows of the table.

Help students see patterns in the 4 numbers; reasoning across the columns and down the columns.

Seconds	Meters
4	25
8	50
16	100
32	200
15	93.75
30	187.50

2. Let's assume that Jace can keep running at the same speed for quite a while.

a. Predict: If Jace keeps going at the same speed (rate), how long will it take him to run 100 meters? How long to run 200 meters?

16 seconds, 32 seconds. Have different students explain their reasoning and compare.

b. Use the graph to check your prediction by stretching the line using the grey dot on the x-axis. Were you right? If not, why not?

Point out the connection between the points on the line and the entries in the table.

c. Fill in the third and fourth rows of the table.

d. Predict: When Jace has run for 15 seconds, how far has he run? What is his distance at 30 seconds?

This question is much more difficult to answer and reasoning down the columns doesn't work as easily. Finding the unit rate or Jace's speed yields the answers more easily. You may want to discuss how the students can "calibrate" the accuracy of their calculated answer by reasoning that the answer for 30 seconds should be a little less than the answer for 32 seconds, which is already in the table.

e. Fill in the fifth and sixth rows of the table.

3. Use the table and the graph to help you describe Jace's run in words and symbols.

a. What is Jace's speed? Show your work.

b. Complete this sentence:
For every second Jace runs, he covers _ **6.25** _ *meters.*

c. Complete this sentence:
To find the number of meters Jace has run, **multiply the seconds by 6.25**

d. Write a formula that expresses the relationship between seconds and meters for Jace's speed.

M = _ **6.25** _ *S (where M is the number of meters Jace has traveled; S is the number of seconds he has gone).*

If your students have difficulty with this, you may want to have them do a transitional equation, which has more English and fewer symbols, such as: The number of meters = 6.25 times the number of seconds

e. Use your formula to fill in the last three rows of the table with numbers of your choice.

4. Take a few minutes to look at connections and patterns in this lesson.

a. Describe patterns that you see in your completed table.

Each entry on the right is 6.25 times greater than its associated entry on the left.

b. Explain connections between the patterns in the table and the formula you wrote in 3d.

The formula M = 6.25S shows the connection between the two columns.

5. Write formulas for the following tables, assuming a constant speed for each table.

a.

Seconds	Meters
100	300
115	345
200	600
4	12
10	30

Formula:

M = 3S

b.

Seconds	Yards
4	20
8	40
10	50
13	65
17	85

Formula:

Y = 5S

c.

Seconds	Feet
10	35
11	38.5
12	42
13	45.5
14	49

Formula:

F = 3.5S

Group work/
Homework

Run, Jace, Run: Revisited

Jace ran with two friends of his, Luke and Mica. Jace was the fastest, and Mica was the slowest. The tables, formulas, and graphs below represent the three runners, but they are all jumbled up. At the bottom of the page, there is a space for you to match them correctly. Below each runner's name, fill in which table, formula, and graph represent that runner.

Table A

Seconds	Meters
10	25
20	50
40	100
80	200

Table B

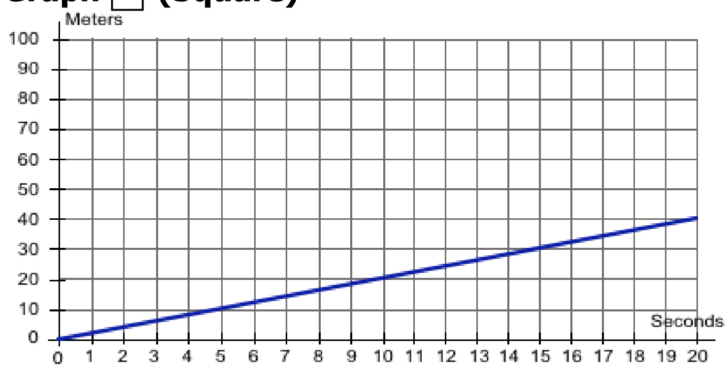
Seconds	Meters
5	10
10	20
20	40
25	50

Table C

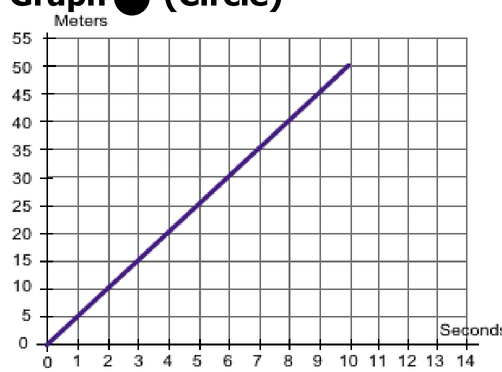
Seconds	Meters
5	25
10	50
20	100
40	200

Formula 1: $y = 5x$ **Formula 2:** $y = 2x$ **Formula 3:** $y = 2.5x$

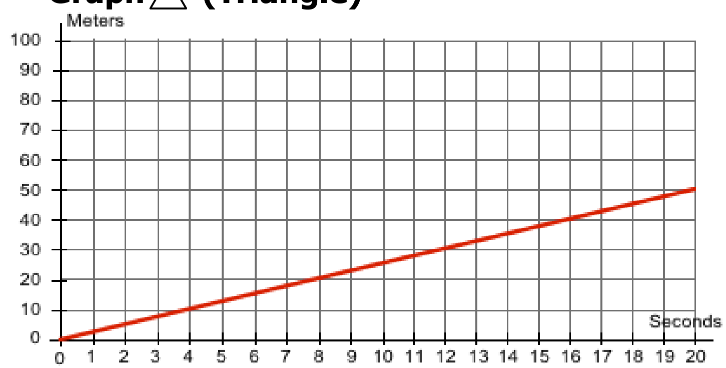
Graph □ (Square)



Graph ● (Circle)



Graph △ (Triangle)



Your answers:

Jace	
Table	C
Formula	1
Graph	●

Luke	
Table	A
Formula	3
Graph	△

Mica	
Table	B
Equation	2
Graph	□

Lesson Plan Day 4

Activities

Back at the Office	Whole class discussion/Group work

Big idea

Students extend the big idea from lesson 3—proportional quantities represented by formulas, graphs and tables—to new contexts. Here the contexts are discrete, involving mostly number of items and cost. All the costs are units.

Notes

Back at the Office 1—whole class discussion

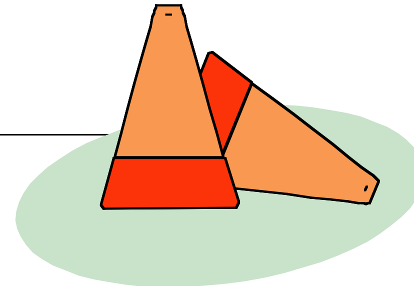
- Begin the lesson by showing the simulation and graph of cones, in MathWorlds file, office1.mw. Read the "to do" and ask students to explain the simulation and graph.
- *Ask: How is this the same or different from the dashes we have been working with*?
- Elicit: time/number items; distance/$; cost/speed. Can run half a meter, can't usually buy half a cone.
- Put the table from the workbook on the board, go through the questions, eliciting table entries as you go. Elicit alternate strategies as much as possible.
- Guide students from graph, to table, to formula. We use the term "formula" to describe the symbols that represent a function, as opposed to table or graph.

Back at the Office 2, 3—group work

- Have students work in groups. Circulate and ask about connections to running problems of previous days.
- See notes on student pages for specifics.

Whole class discussion

Back at the Office



Managing the team keeps you busy. Back at the office, you have a whole stack of tasks to do. Use what you have learned to take care of some "to do's," below.

1. To do: Buy 50 soccer cones.
Use office1.smw to help you answer the questions.

WEEKLY SPECIAL
Soccer cones: $9.60 per dozen. We can break up packages and send as many as you want, at the same rate.
Call now: 1-881-SOCCER

a. Fill in the table to find out how much the different quantities of cones will cost. Choose two of your entries and describe how you got the cost.

The numbers make it difficult to use doubling or halving strategies to complete the table. The unit rate of $0.80 per cone will be useful.

Cones	Cost in $
1	**$ .80**
10	**$ 8.00**
12	$ 9.60
18	**$14.40**
24	**$19.20**

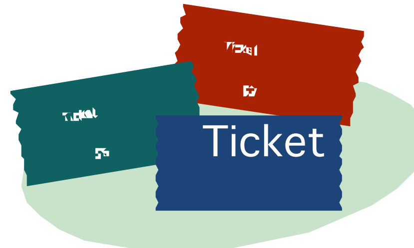
b. How much would 30 cones cost, at the same rate?
Explain your reasoning.

- **30 cones at 80 cents per cone is 30 × 80 = 2400 cents = $24.00 OR**
- **If 10 cones cost 8 dollars, then 30 cones cost three times as much: 24 dollars OR**
- **Since we know the cost of 12 and 18 cones, and 12 + 18 = 30, we can add those costs to get the cost of 30 cones.**

c. Write a formula that relates the number of cones to the total cost, for any number of cones you might want to buy.

Let C be the number of cones; T be the total cost. Then for any number of cones, the total cost is T = 0.80 × C. Students may not produce this complete an answer, but you should help them aim toward it. Again, it may be easier to have students try and write a transitional equation, such as "Total Cost = 0.80 times the number of cones".

2. To do: Set a new ticket price for games. Use the file office2.smw as needed.



a. Fill in the tables to get an idea of how much money the team can bring in for ticket prices of $1.00, $3.00 and $5.00. Ticket sales last year ranged from 20 to 100 per game.

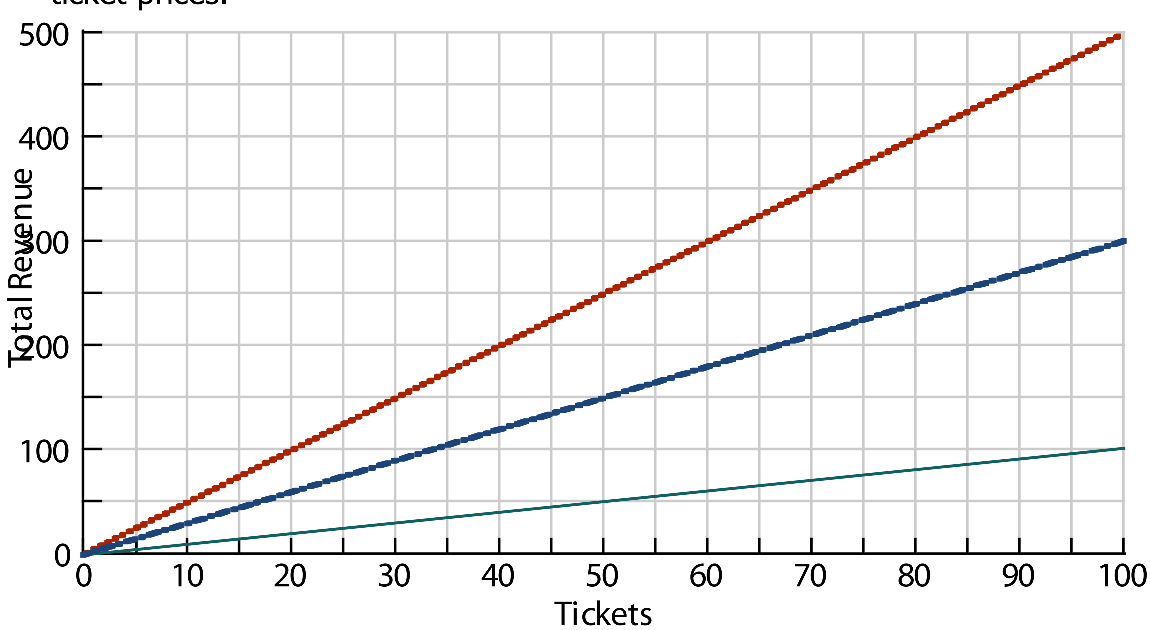
Number Tickets	Total $
20	20
40	40
60	60
80	80
100	100

Number Tickets	Total $
10	30
20	60
40	120
80	240
100	300

Number Tickets	Total $
25	125
50	250
75	375
100	500

Students can use any numbers in the ticket columns.

b. Make a graph so it is easy to compare dollars taken in and tickets sold for the 3 ticket prices.



c. How many tickets would have to be sold, at each price, to take in 200 dollars? Estimate from the graph and then find the exact number of tickets.

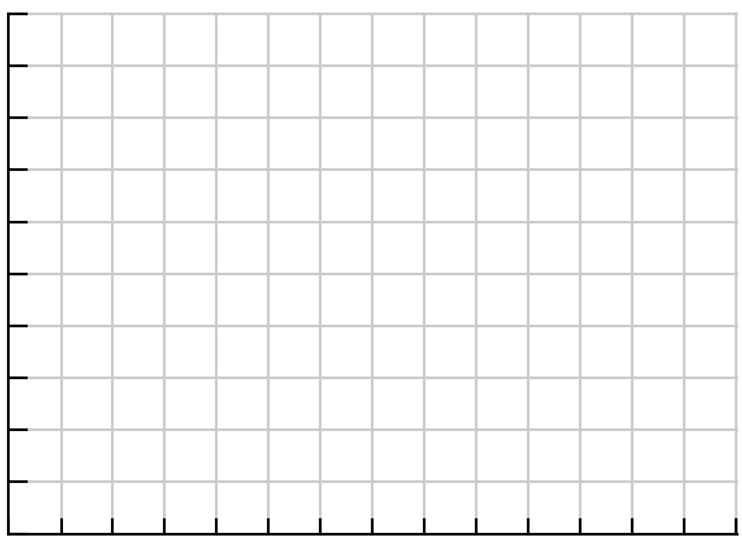
Price	Graph estimate	Exact number tickets
$1	200	200
$3	65	67
$5	40	40

3. Another "to do":

a. Make up your own "to do" using the formula $y = 2.50x$.

Check answers to ensure that the necessary pieces are here. You can use some of the student-written problems as homework. Help students see that the context can be stripped away.

b. Model it with office3.smw. Sketch the graph below.



Lesson Plan Day 5

Activities

Slope and Rate	Whole class discussion

Big idea

Students have discovered and used the idea that steeper lines indicate faster dashes; steeper lines indicate more expensive prices in distance/time and cost/amount graphs. This lesson helps them establish, quantitatively, that in such "accumulation graphs," the slope of the line gives the rate of accumulation. Slope triangles are used.

Notes

Slope and Rate—whole class discussion

- Use 1 to introduce slope triangles and slope as a number describing the steepness of a line.
- Replicate the graph in 1 in MathWorlds or display on overhead or board.
- Have each student make a new line on the graph, then do the slope triangles for his/her own graph. Draw some of the student lines on the board, showing the slope.
- You can use MathWorlds files to find the slope of many different graph lines, until students understand the process.
- Do enough of 2 together so that you can elicit the meaning of slope in the graph.
- In a graph of distance vs. time, the slope of the line of an object moving at a constant speed is equal to the speed of the object. More generally, in graphs comparing the units of a constant rate, the slope of the linear graph equals the rate.

Slope and Rate—group work

- Assign graphs where the students can practice finding the slopes. This can be done using the activities and graphs from previous activities.

Slope and Rate—homework

- Each student should make a poster. (Two pieces of paper taped together is big enough.) At the very least, the poster should show how to calculate the slope of a line. But the real challenge is showing the slope/rate connection visually. You can display those that are mathematically accurate.

Whole class discussion

Slope and Rate

You used unit rates in the previous activities:

You found the speed of runners. *Meters per one second* is a unit rate.

You used the cost of one item to make formulas, graphs and equations. The cost of one item can be written as a unit rate: $3 per ticket, for example.

KEY IDEAS: Rate and Unit Rate

- A rate compares quantities through division.
- "Meters per *one* second" and "dollars per *one* cone" are unit rates.
- $9.60 per 12 cones is not a unit rate.
- "Unit" refers to one.

You learned that for graphs comparing distance and time, or cost and number of items:

The faster the speed, the steeper the line.

The more expensive the item, the steeper the line.

KEY IDEA: Slope

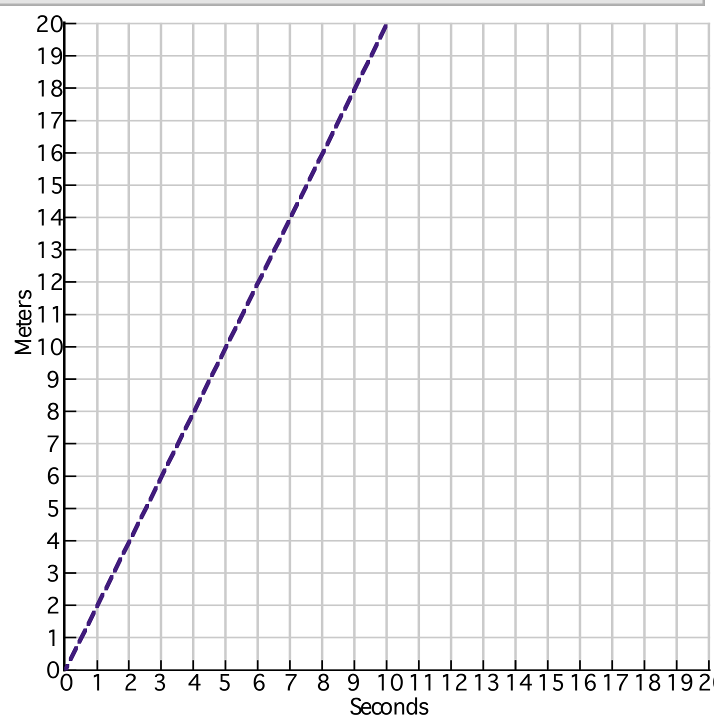
Slope is a number that describes the steepness of a line.

Discuss the above, ask students to explain what it means. Ask for examples of unit rates they have used in life.

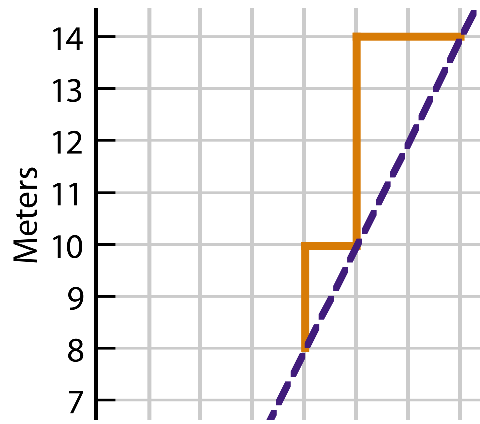
1. Do the following to learn about slope.

a. What is the speed of Purple Girl (dashed line)? How do you know?

2 meters per second; students can use the methods they have learned in previous activities.



b. Make "stairsteps" on the graph of different sizes. A "stairstep" takes you from one point on the line to another point on the line with a horizontal move and a vertical move.



This illustration shows only stairstep of 2/1. Draw 4/2 also. ***Use a very slow walker to develop an example with a new linear graph that has a "shallower slope" and draw some slope triangles for it.***

c. For each "stairstep," record the vertical and horizontal distance in the table. What pattern can you see in the table? Be sure to include the step with horizontal distance of 1.

Vertical	Horizontal
	1

Point out that each entry in the table is in ratio 2 to 1. Each vertical distance represents a change along the meter axis. Its associated horizontal distance shows the associated change in seconds. Each stairstep represents the speed of the runner, 2 meters/second, and also shows how steep the line is, vertical change/horizontal change.

More on Slope

The slope of a line is a number that describes its steepness.

To find the slope of a line:

- Pick two points on the line.
- Find the horizontal and vertical distances between the two points.
- Divide the vertical distance by the horizontal distance.

d. Using the axes on the previous page, draw a green line for Green Runner who went more slowly than Purple Girl. Make a stairstep table for that runner's line below.

Vertical	Horizontal
	1

Give students a chance to do this on their own in the midst of this teacher-led lesson.

e. What is the slope of Purple Girl's line? What is the slope of Green Runner's line?

The slope of Purple Girl's line is 2. The slope of Green Runner's line will vary but must be less than 2.

f. What is the speed of Purple Girl? What is the speed of Green Runner?

Purple Girl's speed is 2 meters per second. The speed for Green Runner varies based on the slope of the line but will be less than 2 meters per second.

2. Go back to some of the other graphs in this workbook and find the slope of the lines.

The other graphs do not have the same-scaled axes. Guide students through a couple, pointing out the issues with changing scale, then assign two for them to do on their own.
Elicit this: the slope gives the rate of the runner, price, etc.

3. Create a poster that describes and explains the connection between rate and slope. Sketch an idea for it below.

This gives students a chance to visually demonstrate how to calculate the slope, and show the rate and slope connection.

Lesson Plan Day 6

Activities

On the Road	Whole class discussion
Road Trip Records	Group work

Big idea

Using all they have learned so far, students investigate another motion context: miles per hour, buses and vans. Now each vehicle travels at different speeds during the same trip, so that the position graph is made of two or more line segments representing the different rates.

Notes

On the Road—whole class discussion

- This activity calls for correlating a graph and a simulated motion, as in “Race Day,” but this time, it’s a bus and van trip.
- Work through interpreting graphs of motion with narratives, using the MathWorlds files to verify the story. (The step button helps.)

Road Trip Records—group work

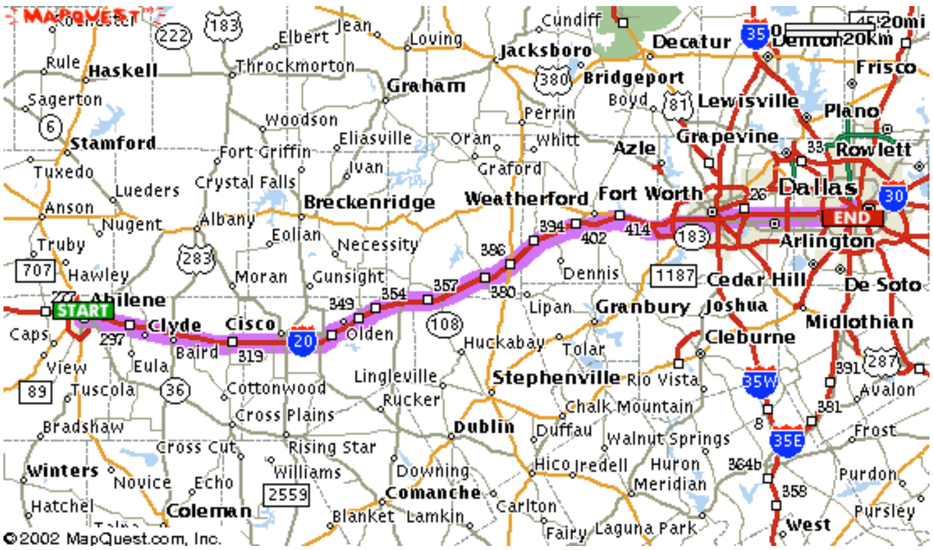
- Problem 1 is a direct extension of “On the Road.”
- The rest of the activity asks students to model stories with graphs and simulations. You can guide students through problem 2, if you think the leap into this new activity will be too much for your students.

On the Road

Every year the team makes the trip from Abilene to Dallas, for a special challenge match. They take both a bus and a van on the trip to accommodate all players and team boosters.

The good news: They have won the challenge match for many years in a row.

The bad news: The trip often has troubles—breakdowns, traffic tie-ups, you name it!



Abilene to Dallas Map

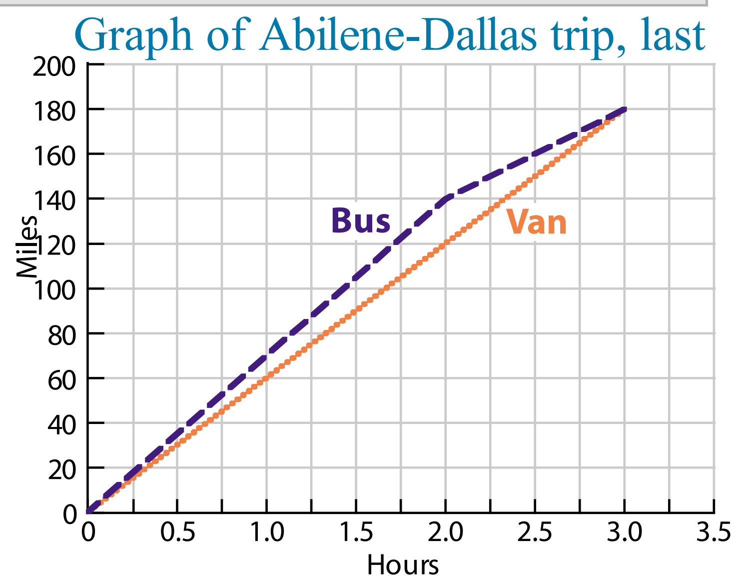
1. What information can you get about the trip from looking at the map above?

> This is a truly open-ended question. Ask for a quick explanation of each answer, get as many as possible. Examples:
> - The distance is about 180 miles (use scale or add up numbers).
> - The travel is from west to east.
> - The road is fairly straight, but not "beeline."
> - The towns we go through on the way.
> - The highway number.

2. Use the file, onroad2.smw, to see what happened on last year's trip. Look at the simulation and the graph. Use the step button as needed so that you understand what happened.

Run the simulation a few times, with and without step. Correlate each segment with a vehicle traveling.

a. What information can you get about last year's trip from looking at and analyzing the graph? (Write down everything you can think of, and be sure to include the speeds of the vehicles.)



The information from the graph is different than from the map. Examples:

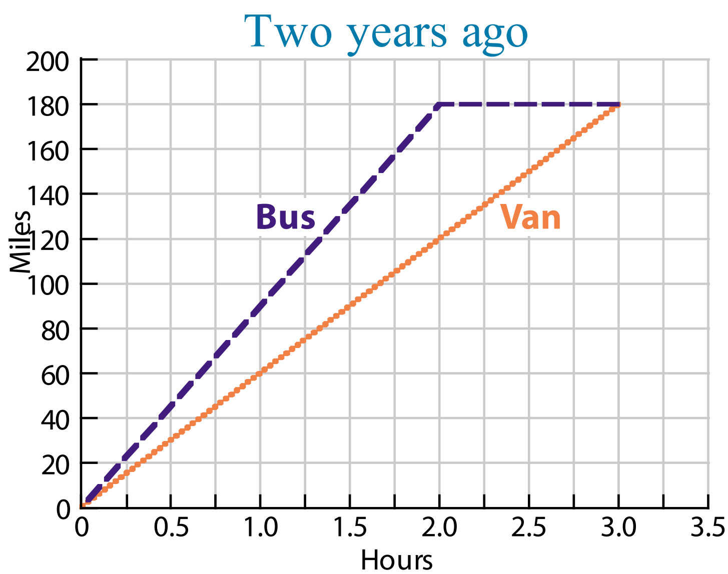
- **The time taken for each vehicle; the distance traveled. The speed of each vehicle.**
- **Time: 3 hrs Distance: 180 miles**
- **Speeds: Van at 60 mph. Bus at 70 mph, then 40 mph.**

This is the first time students are seeing two rates for the same object/trip. ***Ask: What is different about this graph from previous ones? What does it mean?*** **Remind students that each segment is still just an average speed; start ups and slow downs are not shown.**

b. Write a paragraph that describes the motion of the bus and the van on the trip.

Ask: What could have happened? Encourage creativity.
Get "likely stories," e.g., the bus and van started from Abilene at the same time. The bus went fast, 70 mph, but slowed to 40 mph after 2 hours because of bad traffic. The van kept a steady pace of 60 mph the whole time.

3. The trip from Abilene to Dallas two years ago went fairly well.



a. Predict from the graph: Which vehicle arrived first? How long did it take each vehicle to make the trip?

> ***Ask: What does the "flat" line segment mean?***
> **Elicit: Stayed at same place for 1 hour. Horizontal line has 0 slope: 0/2 mph.**
> **The bus arrived first, after 2 hours. The van took 3 hours.**

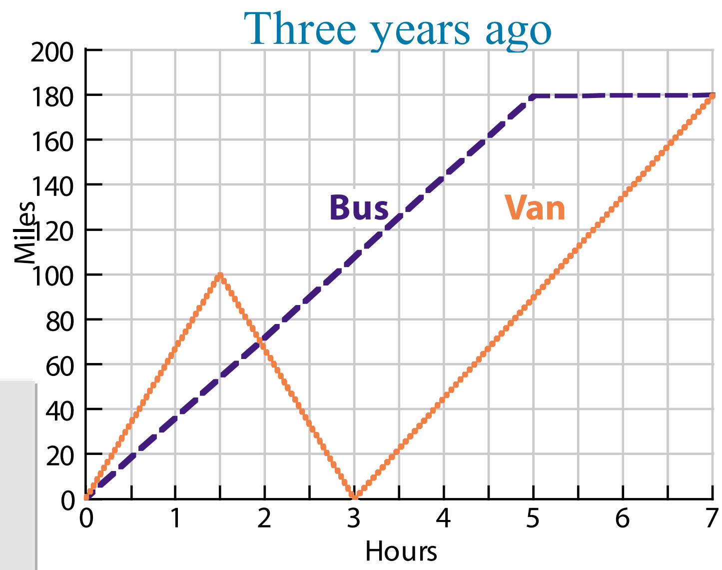
b. Run the simulation in the file onroad3.smw. Were your predictions right? Explain any differences.

> **Run the simulation a couple times, also use step. Correlate each segment with a vehicle traveling.**

c. Tell a story that could go along with the graph of two years ago.

> **Encourage a variety of stories. Ask others to verify accuracy and explain.**

4. Three years ago, the trip was not so smooth. Here is the graph of the bus' and the van's travel on that trip.



a. What did the van do after traveling for one and a half hours?

This is the first time students see a negative slope in this unit.
Elicit meaning: It went backwards; it started back toward Abilene. It went from 100 miles out to 0 miles (start point) over a 1.5 hr segment.

Depending on your class, you can calculate the negative slope and talk about velocity versus speed. Velocity is speed with direction. You can also simply leave this with the interpretation of "went back."

b. What happened here? Tell the story of this trip.

Encourage creativity, step through the simulation to check correctness.

c. Use onroad4.smw to verify your prediction and story.

5. Think about the mathematics you did with the soccer players' dashes and the mathematics you did with the bus and van trips. Write a sentence or two explaining the similarities and differences between the two situations and the mathematics you did with each.

In common:
- ❖ Graphs of distance vs. time
- ❖ Slope shows speed

Different:
- ❖ Rates of same moving object

In the next activity, students will model stories of bus and van trips with graphs. If you think they may need help with this, you can do one with them first.

Group work

Road Trip Records

Students extend their thinking and consolidate their learning in this activity.

1. There is one more travel record in the files for the Abilene–Dallas trip.

a. Use MathWorlds to explore the simulation of the trip.

- Open the file roadtrip1.smw.
- Run the simulation (Note: the graph is hidden on purpose).

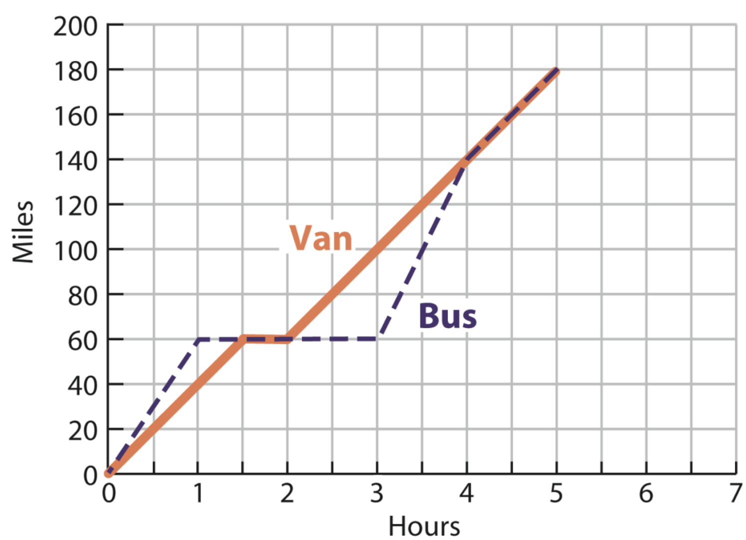
b. Describe the bus trip and the van trip in a few sentences.

Bus: Bus traveled at steady rate for an hour, stops for a while, then goes again.

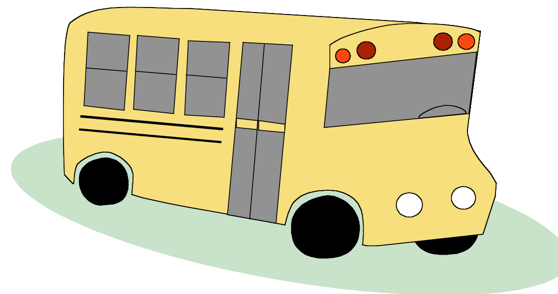
Van: Van traveled at a steady rate, stopped where the bus was stopped, but not for as long, then steady to Dallas (180 mile marker).

c. Predict what the graph will look like by making a sketch of the van and bus trip. Verify by checking points on the graph and playing the simulation.

In students' predictions, look for relative rates, and horizontal lines to indicate stopping.



2. One old-timer bus driver told about her worst trip from Abilene to Dallas:



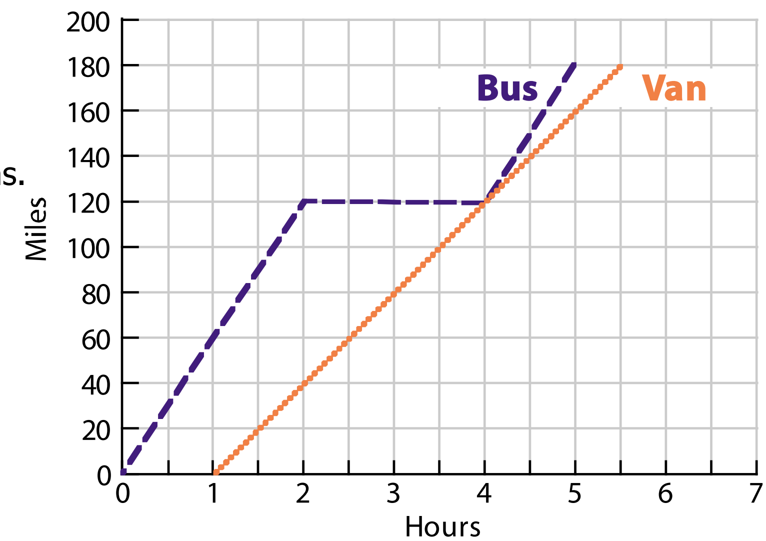
"The team had to take a bus and a van from Abilene to Dallas, once again. The bus left first and traveled at 60 miles per hour (mph). The van left an hour after the bus, and poked along at 40 mph, because of road construction.

"After two hours, the bus broke down and stopped. The bus just sat there for an hour while I looked at the engine. I decided a new fan belt would get us going. (Meanwhile, the van was still traveling at only 40 mph.)

"I was lucky: a call to TAAA got a fan belt delivered and installed in one hour. We got the bus back on the road and kept going at 60 mph. The van kept going at 40 mph.

"I remember all those details like it happened yesterday, but I sure don't know who got there first. And where was that van when we broke down, anyway?"

a. Sketch a graph showing the trip she described.



b. Answer the driver's questions.

Who got there first?

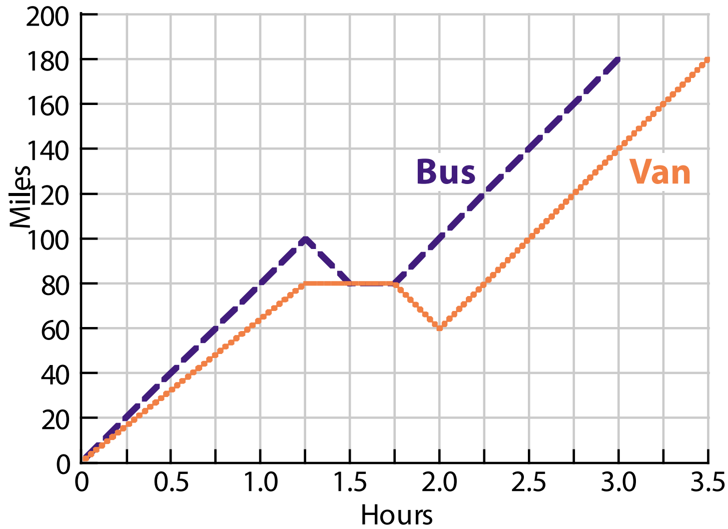
The van got there first.

Where was the van when the bus broke down?

The van was only at the 40 mile marker when the bus broke down. Some of the students may wonder what happened when the bus and van were at the same point, as shown by the intersection point at (4, 120). You can also raise this issue as a way to emphasize the meaning of intersecting lines.

c. Now use roadtrip2.smw to model and graph the trip.

3. Here is another graph of an Abilene—Dallas bus trip from the past.



a. One hour into the trip, which vehicle had gone farther?

The bus.

b. When were the vehicles at the same place, at the same time?

At the beginning, and between 1.5 and 1.75 hours, when they were both at 80 miles.

c. Which vehicle went back toward Abilene?

Both vehicles went back toward Abilene during the trip. The bus between 1.25 and 1.5 hours, and the van between 1.75 and 2 hours.

d. When were the vehicles going at the same speed at the same time?

Between 1.5 and 1.75 hours, both vehicles were going 0 miles per hour.
Between 2 and 3 hours, both vehicles were going 80 miles per hour.

e. Pretend you were on the trip. Tell the story of the trip: what happened to the van and the bus?

- **The bus and the van started at the same time, with the bus going a bit faster.**
- **At 1.25 hours, the van broke down and called the bus, and the bus turned back to help the van.**
- **Between 1.5 and 1.75, hours the bus stopped with the van while the bus driver tried to help fix the van.**
- **At 1.75 hours, the van was fixed enough to drive, but needed to go back into a town to pick up a part to be fully fixed. The bus continued on towards Dallas at 80 mph.**
- **At 2 hours, the van did a "drive by" pickup of the part and installed it in a flash (since it did not stop!), and then turned back towards Dallas at 80 mph.**
- **The bus arrived in Dallas half an hour before the van.**

Lesson Plan Day 7

Activities

Graphs of Motion	Group work

Big idea

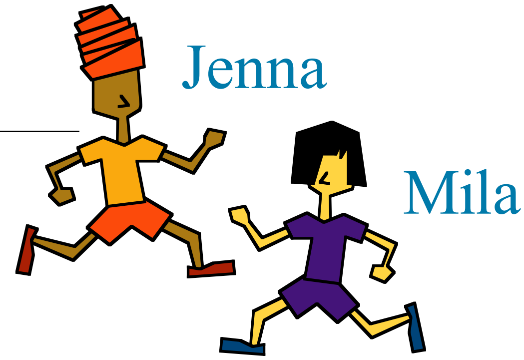
This activity provides practice in motion contexts, including single and multiple rate motions, and coordination of graphs and stories. Students consolidate their learning by making 4 “model” graphs, stripping away particular contexts of runners or vehicles.

Notes

Let students work in groups on the problems for the first half of class. Then either go over their answers for item 3 together, or do 3 together, depending on how much progress they have made. Compare different graphs that students made by drawing them in different colors on the board or overhead or by modeling them on the same axes in MathWorlds.

Group work

Graphs of Motion

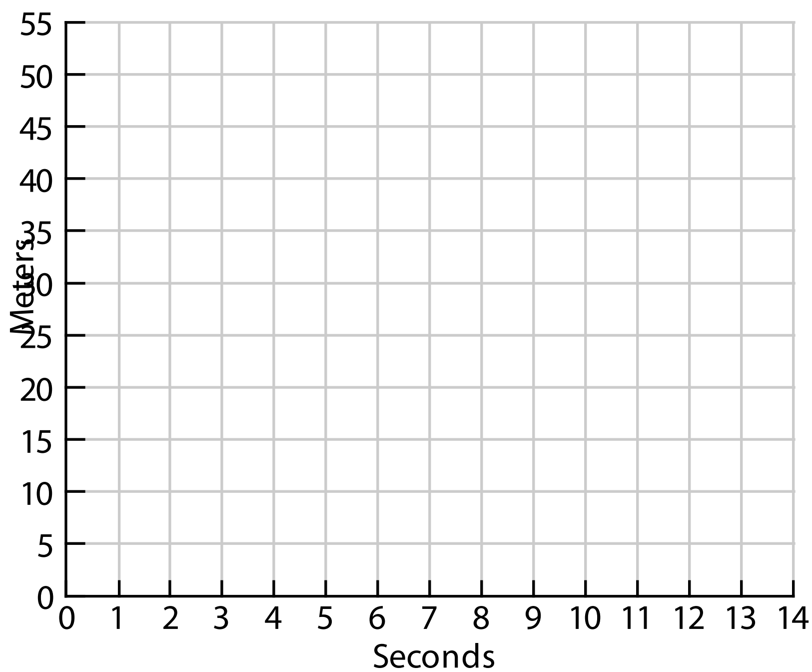


Interpreting and creating graphs are both important skills in mathematics, in and out of the classroom! The following activities give you a chance to practice.

1. Sketch a graph for each of the following 50-meter race stories. Then make your graph in MathWorlds, and run the simulation to see if it fits the story.

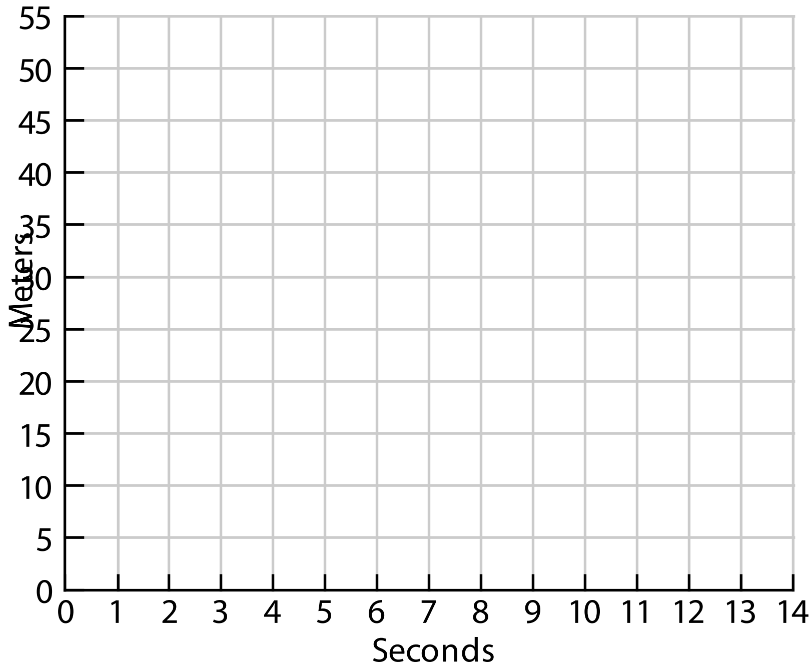
Different graphs can fit these stories. See MathWorlds files for examples.

a. Jenna ran faster than Mila for the first 25 meters. Then Jenna twisted her ankle and fell at 25 meters. She couldn't get up. Mila slowed down, but she kept running across the 50-meter line. Use graphrace1amain.smw to build and check your solution. Use graphrace1a.smw to see an example of a possible solution.



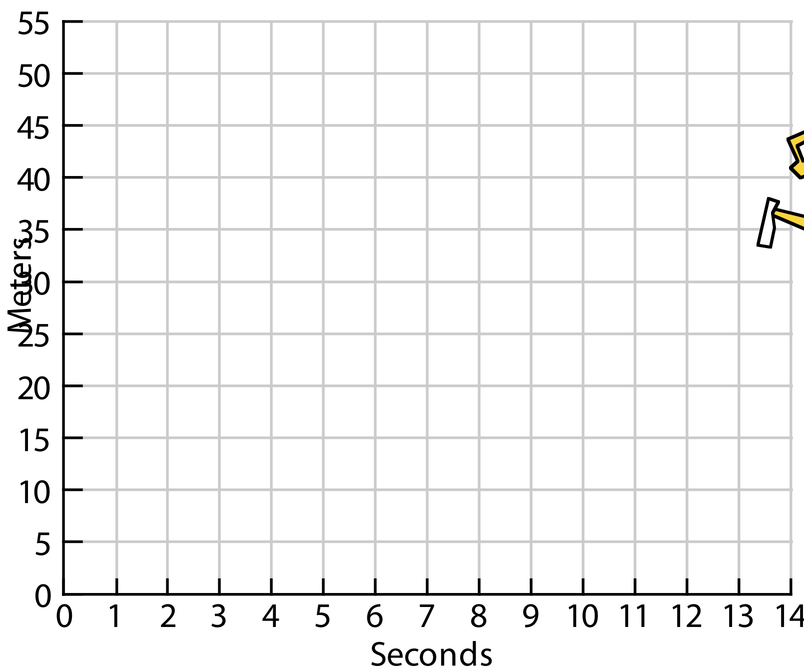
Jenna and Mila both start at (0,0). Jenna's line must be "above" Mila's line until Jenna arrives at 25 meters. Then, Jenna's line is horizontal on the 25 meter mark for the rest of the race. At the time Jenna falls, Mila's line must get less steep (smaller slope value) than it was before Jenna fell.

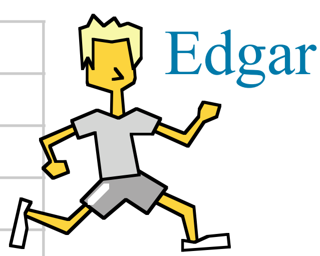
b. Chanda and Shawntee were neck and neck for the first 30 meters. Then Chanda sped up and passed Shawntee, winning by 2 seconds. Use graphrace1bmain.smw to build and check your solution. Use graphrace1b.smw to see an example of a possible solution.



Both girls' graphs start at (0,0). Until they arrive at 30 meters, both lines must coincide (be on top of each other). At 30 meters, Chanda's line's slope increases. The endpoints of the lines are 2 seconds apart on the x-axis.

c. When Duane and Edgar raced, Duane was so sure he would win that he gave Edgar a 10-meter head start. Each boy ran at a constant rate the whole race, and Duane did win. Use graphrace1cmain.smw to build and check your solution. Use graphrace1c.smw to see an example of a possible solution.



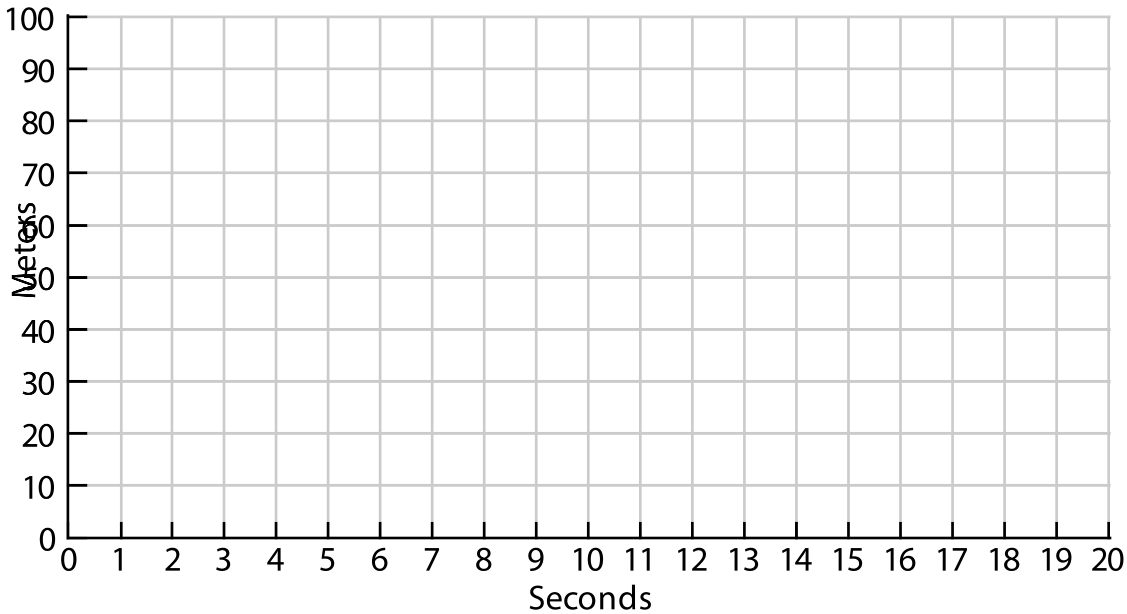


Edgar's line begins at (0,10). Duane's line at (0,0). Duane's line crosses Edgar's before the end points. Each is a single line segment, same slope.

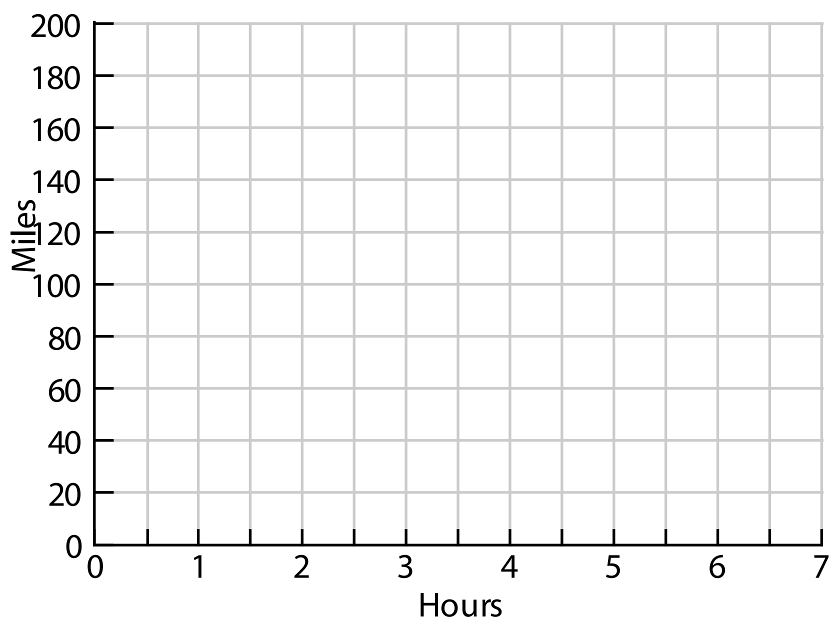
2. Watch each simulation and then sketch a graph on the axes provided.

Check graphs against files graphrace2aWithGraph.smw and graphrace2bWithGraph.smw in the teacher package of MathWorlds files.

a. graphrace2a.smw



b. graphrace2b.smw

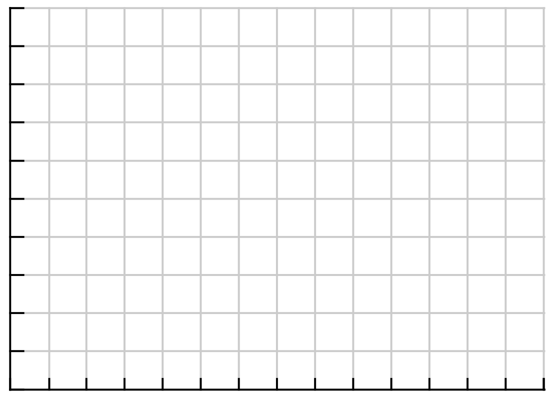


3. In order to quickly understand graphs, it helps to know some "by sight." Sketch graphs for each of the following situations, so that you will be able to recognize them in future work.

a. Standing still.

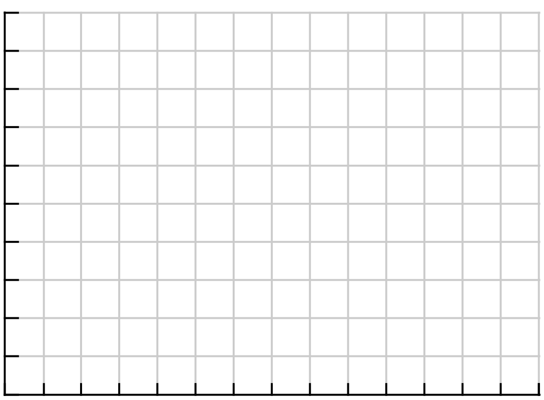
Any horizontal line.

b. Going one rate, then going at a faster rate for the same amount of time.



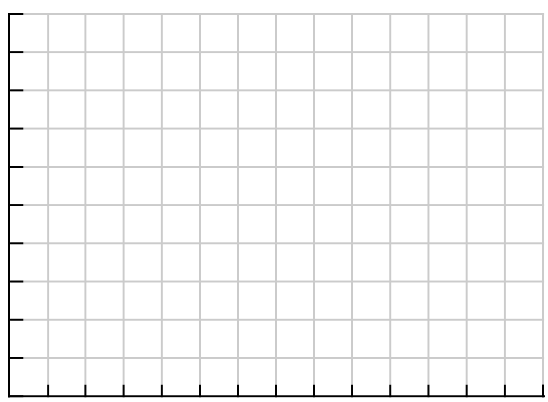
A graph with two contiguous line segments, each covering same distance on x-axis, greater slope for second segment.

c. Going one rate, then going at a slower rate for the same amount of time.



As in (b), but smaller slope for second line segment.

d. Going forward and then backward at the same rate.



Two contiguous line segments, first with positive slope, second with negative slope, same absolute value.

Lesson Plan Day 8

Activities

Salary Negotiations	Whole class discussion
Summer Job Advice	Group work
All about MPG	Homework

Big idea

Students explore multi-rate graphs in a non-motion context: dollars earned per hour. The same concepts of *varying slopes* and *connections among words, graphs, and tables* are used as in previous lessons. Problem solving is emphasized.

Notes

Salary Negotiations—whole class discussion

- Lead students through these activities, reminding them of the similarities to “Road Trip Records” but also pointing out differences.
- Use hired.mw to help model the problem, so that students will feel comfortable modeling problems on their own later.
- Explain how the “world” in hired.mw works: how it uses bars to represent accumulating amounts.

Summer Job Advice—group work
These problem solving activities are not as challenging as those in “Salary Negotiations” but provide opportunities for students to use similar approaches on their own. Help students make connections to the “speed” problems they have done.

All about MPG—homework
This homework prepares students for the next day’s lesson, provides some information about miles per gallon and fuel efficiency, and has students practice calculations.

Whole class discussion

Salary Negotiations

Read the memos and answer the questions.

To: Acting team manager
From: Personnel office

If you are hired as official manager, you will need to choose a pay scheme. Your season lasts 10 weeks. You have two options for getting paid:

Double Overtime:
Up to 100 hours—$7.00 per hour
Any hours over 100 in the season—$14.00 per hour
OR
Same Wage:
$10.50 per hour, no matter how many hours worked.

Please let us know which way you prefer, as soon as possible.

To: Acting team manager
From: Personnel office

One more payment option: We could also pay you a flat fee.

Flat Fee:
You get $750 for the season, no matter how many hours you work.

Please let us know which of the three ways you prefer, as soon as possible.

1. What should you consider before you decide?

How many hours you think you will work.

2. Remember that the season is 10 weeks long. Which pay option will earn the most money if you work, on average, 10 hours a week? 20 hours per week?

10 hours a week means 100 hours all season, so you would get no overtime, and earn $700. So it is better to choose the "same wage" option.
At 20 hours per week, you work 200 hours for the season, getting overtime rate for the second 100, so $700 + $1400 = $2100. The "same wage" rate will yield $2100, so it doesn't matter which option you pick for 20 hours per week.

3. Compare the two pay options for any number of hours you might work for the season. Use graphs, tables, or formulas.

See lesson plan.

4. Now compare and discuss the three payment options from *both* memos.

Group work

Summer Job Advice

You have figured out the best pay option for your job. Now give advice to these jobseekers. You may use the MathWorlds file summer.smw, to help you with each problem, or just use paper and pencil. Do not worry about taxes or other paycheck withdrawals for these problems. (They will have to figure out that part for themselves.)

1. Kim wants to earn money for school clothes for next fall. She is going to baby-sit for the Santiago family this summer. She will care for their children 2 days a week, for 4 hours each day, for a total of 6 weeks.

She can get paid one of two ways:

$7.00 an hour

OR

a one-time payment of $350.00 for the whole summer

a. Which pay scheme will give her the most money?

According to the current schedule, Kim will work 48 total hours.
48 hours at $7.00 per hour is $336.00. She will make a little more money with the one-time payment, if they don't ask her to work any additional hours.

b. Kim isn't really sure the Santiagos will need her the same number of hours each week. Make a graph or table that compares both ways of getting paid for 0 to 60 hours.

The graph should show a horizontal line at $350.00, then a line with a slope indicating $7.00 per hour.
The intersection point is at 50 hours, so if she works more than 50 hours for the summer, she will come out ahead with the hourly rate.

Compare with bus trip/racers:
Time is still on the x-axis. Point out that this is a very common convention.
"Dollars earned" has replaced "meters traveled."
In this case, only single rate lines are used.
The slope is the rate of pay, rather than speed, the rate of travel, but the slope still shows a rate.

2. Marta has two summer job offers:

helping in her aunt's store for $8.00 per hour

OR

being a preschool aide for $6.50 per hour

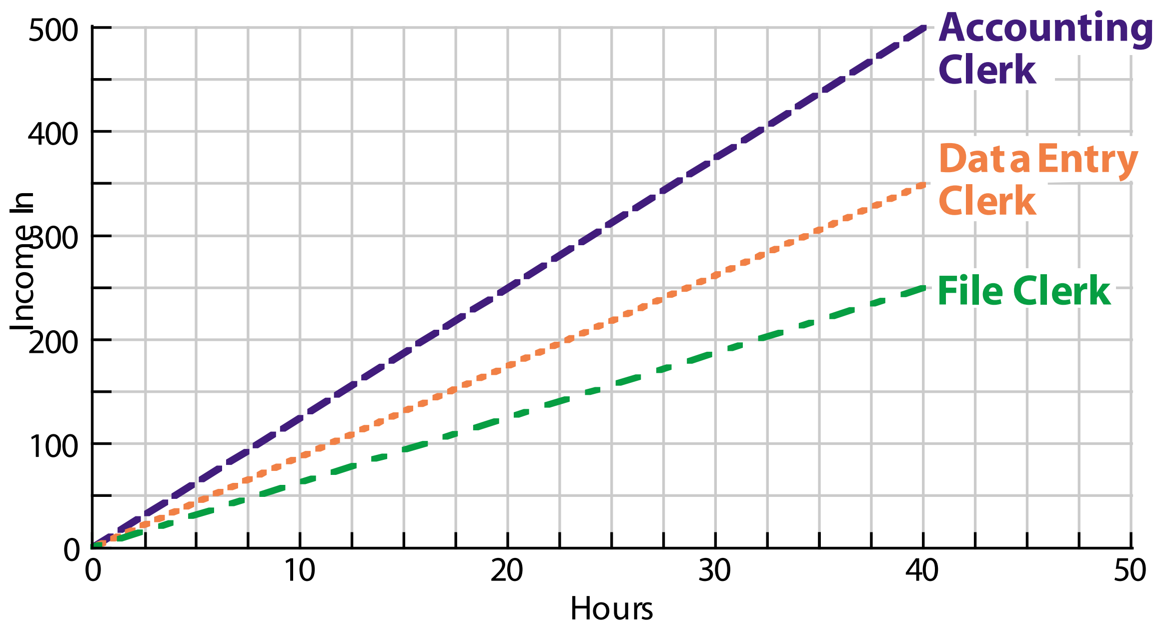
Each job lasts 6 weeks and is for 20 hours per week. She really cannot decide which job to take: she wants to help her aunt and the pay is better, but she loves working with small children and thinks the preschool job could look good on her college applications.

Make a graph so Marta can compare the difference in total pay for 0-150 hours.

The graph again has only single rate lines. Simply producing the graph is all that is called for.

A possible homework problem: ask students to write Marta a note, persuading her to take one job or the other, using the graph to support it.

3. Tony applied for work through SummerHelp, an agency that helps place teens in summer jobs. They gave him a graph showing the earnings of several kinds of jobs:



a. Make a table for Tony showing how much he would earn for 10, 20, 30, and 100 hours of work in each job.

Students may:

- ❖ **Estimate from the graph.**
- ❖ **Use points on the graph that are easy to read to fill in the rest; see 40 hours below. (Because the rates are constant, the kind of reasoning they did with tables in "Run, Jace, Run" will work.)**
- ❖ **Calculate the rate of pay per hour and use that.**

Hours	Accounting clerk	Data entry clerk	File clerk
10	125	87.50	62.50
20	250	175	125
30	375	262.50	187.5
100	1250	875.00	625
40	500	350	250

b. Tell Tony how much each kind of job pays per hour.

Accounting clerk	**$12.50 per hour**
Data entry clerk	**$8.75 per hour**
File clerk	**$6.25 per hour**

4. Trevonne needs to make at least $1000 this summer. He can make $6.50 an hour working at the donut shop, working as a lifeguard pays $7.75, and the computer store offers $9.25 an hour.

a. How many hours would Trevonne have to work at each job to earn $1000? Show calculations, formulas, graphs, or tables that you used to get your answers.

Answers will vary, of course. Here is the formula method, which will not be the most likely method for 7th graders:
E = 6.50H where E is the total earnings in dollars and H is the number of hours worked.
1000 = 6.50H. Substitute 1000 for E.
1000/6.50 = H. Divide both sides by 6.50.
He needs to work almost 154 hours to earn 1000 dollars at the donut shop.

This problem provides general problem solving practice using rates.

The jobs are in different parts of the city. Trevonne can walk to the donut shop in 5 minutes. To get to the beach for lifeguarding, he can ride with his friend, Jana, and it will take 30 minutes each way. The computer store is three bus rides away, for a total of an hour's travel time each way. Trevonne wants to spend the least time possible working (including commute time) this summer.

b. Consider the commute times and the working hours, and then give Trevonne advice about which job to take. (Assume each job requires an 8-hour day.) Again, show your work.

You can skip this if time is short. The concepts of the unit are not extended in this problem, but it is a good problem solving activity.
One way to reason:

$ per hour	8 hour earnings	Time at work and commute	Actual $/hr
6.50	52	8.17	6.36
7.75	62	9	6.89
9.25	74	10	7.40

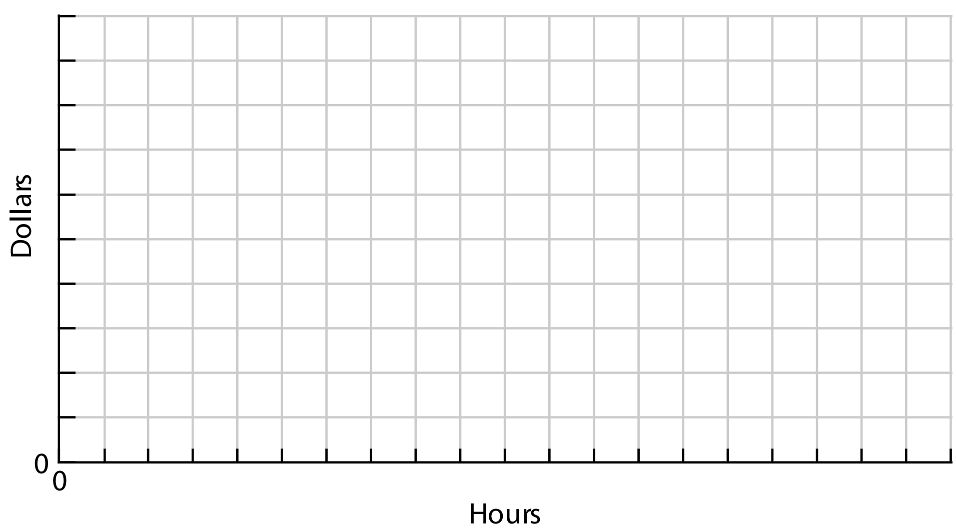
"The computer job still gives a higher rate of earning even when the commute time is added in, but the difference is not as dramatic. I advise Trevonne to take the lifeguard job. He won't earn that much less and he can have more fun than at the computer store."
Qualitative reasoning could support different choices here; there is no one right answer. Note the connection between this and "Isabella Improves," but in this case, the earnings and the time are changing in each row.

5. The YES program helps teens in Amarillo get summer jobs. (See newspaper article on the following page.) If the teens work in a store, YES pays $2.00 per hour of their wages. If the teens work in a daycare center, YES contributes all of their minimum wage: $5.15 per hour.

a. Write formulas for each rate of contribution, relating hours worked to total dollars earned.

C = 2.00H for store.
C = 5.15H for day care, where C is the total YES contribution and H is the number of hours worked.
Here, formulas are used in a new context.

b. Create a graph showing the YES contribution to the salaries of the two types of jobs, for 0 to 100 hours.



Simple graphing techniques from early lessons will work here, graphing from (0,0) to end points of (100 hours, $200) and (100 hours, $515).

c. Write a letter to send with the graph, explaining to employers how much they can expect YES to contribute if they hire a teen this summer.

Look for explanations of how to use the graph.

32 student employees recognized for completing summer jobs

10:43 a.m. CT
By KAREN D. SMITH
Globe-News Feature Writer

Thirty-two teen-agers stand on good career foundations because of 21 businesses and the Youth Excelling & Succeeding Project employment program, project officials said on Aug. 12.

"My father would always tell us you have to have a good foundation, because without a solid foundation, the house is not going to stand," Potter County Commissioner Iris Lawrence said to employers and teen workers gathered for the YES Employment and Training Recognition Luncheon at the Black Historical Culture Center. Lawrence, a member of the YES Project oversight committee since its inception, outlined the beginnings of the program for luncheon attendees.

Funded through the Texas Department of Protective and Regulatory Services Community Youth Development Program, YES provides job, recreation, mentoring and educational opportunities for youths living in North and Northeast Amarillo neighborhoods.

At summer's start, the PRPC placed 43 students in jobs with private employers and nonprofit agencies, according to PRPC Executive Assistant Rebecca Rusk. YES funds pay $2 per hour of the wages for youths placed in for-profit businesses and 100 percent of the minimum-wage salaries for youth placed at public and private nonprofit agencies, Globe-News files show.

Of the 32 students who completed the summer program, 20 were female and 12 male. Collectively, they worked more than 10,000 hours and earned combined wages of $50,000, Rusk said.

Lawrence also shared a personal story of opening doors, referring to her introduction when it was noted at the end of a long list of posts that she had even been a clerk at the former downtown Fedway department store in the 1960s.

"I was the first black clerk employed at Fedway," she said, explaining that the store owner approached her pastor at Mount Zion Baptist Church with a plan. "He told (the pastor), 'I would like to hire a Negro to work in our store.' Most blacks at that time had been messengers, janitors and so on."

"He wanted to open the doors here in Amarillo, Texas," Lawrence said. "It was the early '60s, and it was quite hard for a black to get a decent job."

Later, she was the first black female clerical worker hired by another company in Amarillo, where one black male also was employed. "The two of us opened doors again," she said.

Lawrence told the story, she said, to make a point to the teens. "The door has been opened, so it's time now that the young people take advantage of all you have before you," she said.

Homework

All about MPG

Miles per gallon (MPG) is an important rate when dealing with motor vehicles—cars, trucks and vans. MPG tells you how many miles you can drive on each gallon of gasoline you put into your vehicle.

MPG information from a new car sticker

Vehicle type	Average MPG
Compact cars	26.1
Vans	15.7
Pick ups	17.1
SUVs	17.3

Average MPG for vehicles of different types

1. Compute the fuel mileage, or miles per gallon (MPG) for each situation below:

a. On a recent trip, Sarah's car traveled 339 miles and used 12 gallons of gasoline. What was her MPG for that trip?

339/12 gives 28.25 MPG

b. On Monday, Emily filled up her truck's tank and then drove all over town all week. On Friday, she stopped at the service station again. She used 10 gallons to fill the tank, and noticed that she had traveled 156 miles since the last fill up. What was her MPG for the week?

156/10 gives 15.6 MPG

c. Oscar kept a record for his car. Fill in the MPG column.

Date	Miles traveled	Gallons	MPG
3/02/03	252	12	21
3/08/03	356	11	32.36

2. Fill in the blank: The higher the MPG of a car, the _ less _ money you have to spend on fuel for the car.

Lesson Plan Day 9

Activities

How Far on How Much? MPG	Whole class discussion
Suiting Up	Group work

Big idea

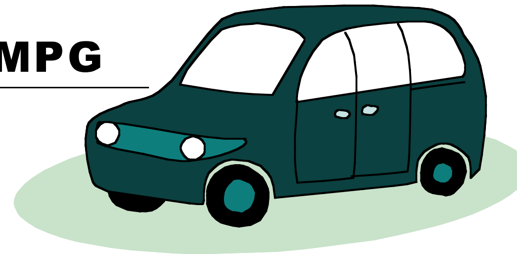
New contexts in which to use the concepts, skills, and techniques learned in the unit.

Notes

Let students work independently first, then go over as a whole class. This is a chance for them to see how well they can use what they have learned. You may have to step in for “Suiting Up,” to help students see the connection to multi-rate graphs.

Whole class discussion

How Far on How Much? MPG



A big expense for the soccer team is buying fuel for the bus and van used for traveling to "away" games. To try to reduce costs, you investigate the miles per gallon that each vehicle gets.

1. Frida, the van driver, is always worried about running out of gasoline on the way home from a game. She keeps a little chart on the dashboard that tells her how many miles the van can go on how many gallons. It has been on the dashboard so long that some of the numbers have worn off.

Gallons	Miles
0	0
2	30
4	**60**
6	**90**
8	**120**
18	**270**
14	210
22	**330**

a. Fill in the chart for Frida.

Note that the numbers in the gallons column are *not* all in descending order with same difference. This requires students to do more than just "add down."

b. How did you know what numbers to fill in?

Look for varying strategies, as in "Run, Jace, Run."

c. According to this chart, how many MPG does the van get? How do you know?

15 MPG is the unit rate that relates the gallon and miles columns.

d. Write a formula that tells you how many miles the van will travel for any number of gallons.

M = 15G where M is miles that can be driven and G is gallons used.

e. Use your formula to determine how many miles the van can go on 4 gallons. Does it match the chart?

> **M = 15(4)**
>
> **Have students use the formula and substitute, as well as other methods they come up with.**
>
> **M = 60 miles**

f. Use your formula to predict how many gallons of gas are needed for a 450-mile trip.

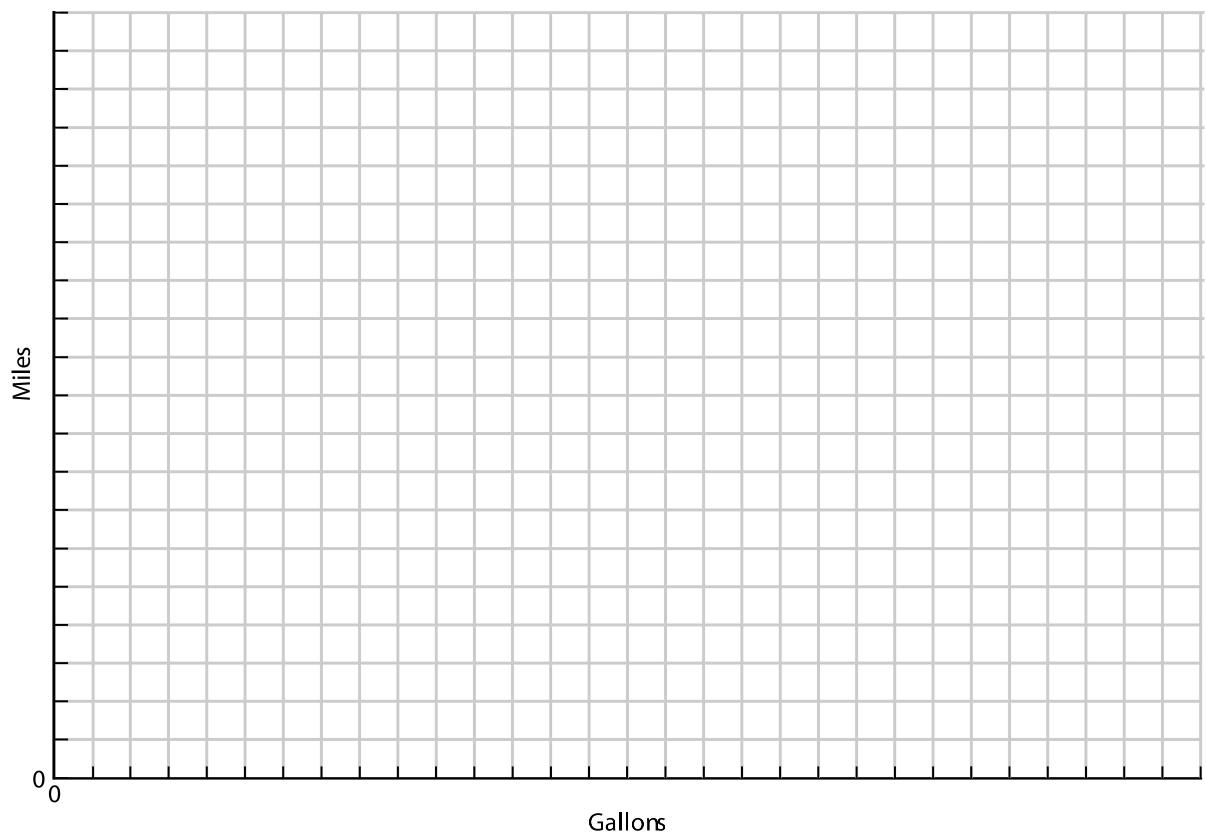
> **450 =15G**
> **G = 450/15**
> **G = 30**

g. Compare the formula and table: What is each good for? What is each NOT good for?

> **The table lets you see at a glance commonly used numbers, as Frida needs. The formula can be used for any number of miles or gallons, to find the corresponding value.**

2. We can use a graph to compare the MPG of the van and MPG of the bus.

a. Make a graph of the van's MPG for 1 to 30 gallons. (Miles on vertical axis; gallons on the horizontal axis.) You can use mpg.smw.



b. The bus gets 9 miles per gallon. Sketch the line of the bus' mileage on the axis above and write a formula that represents the relationship between miles traveled and gallons of fuel used for the bus.

This line will be less steep than the van's MPG line.

c. Write a memo comparing the fuel costs for the van and the bus when they are used to take the soccer team to games. Use this fact: most games are between 50 and 120 miles away from home.

To: School Principal

From: Acting Soccer Team Manager

Re: Fuel costs for soccer team travel.

The bus gets 9 miles per gallon, and the van gets 15 miles per gallon. Since most games are between 50 and 120 miles from home, the bus needs between 5.5 and 13.3 gallons of gas, and the van needs between 3.3 and 8 gallons of gas for each game.

Look for correct mathematics and writing that helps explain the numbers. You can also encourage students to make their own tables to summarize the information.

You have compared the MPG for the bus and the van, assuming that the MPG stays more or less the same all the time. But you know that MPG varies. For example, a vehicle uses more fuel per mile when driven in a city or town than when driven on the highway. This is important for the following problem.

3. Frida the driver calls in from the road: the fuel gauge on the bus is not working, and the team is already late for the Ballinger game. She needs your help to figure out if they can make it to the game without stopping for a fill up.

Here's what you both know:

- The bus gets 6 mpg in the city, and 10 mpg on the highway.
- The tank holds 50 gallons of fuel.
- Frida filled the tank on Monday and has driven 136 miles, around the city, since then.
- The bus has to travel 55 more miles on the highway to get to the game.

Exactly how far can the bus go with the fuel remaining?

Use graphs, tables, computations and/or formulas to show how you found out. (You may use the MathWorlds file, mpg.smw, if you want.)

One way to solve the problem:

50 gallons of fuel on Monday.

$$136 \text{ miles} \times \frac{1 \text{ gallon}}{6 \text{ miles}} \approx 22.7 \text{ gallons used}$$

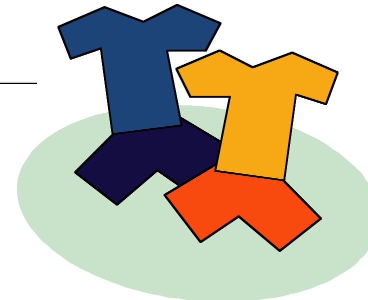
50 gallons – 22.7 gallons = 27.3 gallons remaining

$$27.3 \text{ gallons} \times \frac{10 \text{ miles}}{1 \text{ gallon}} = 273 \text{ miles}$$

There is plenty of fuel to get to Ballinger.

Group work

Suiting Up



The team's uniforms are looking raggedy and you have some new team members. Time to buy new uniforms! Through the Internet and ads in soccer magazines, you have put together a fair amount of information. Some mathematics will help you find the best deal.

Use the MathWorlds file, suitingup.smw, to help you as needed in the following problems.

1. Uniforms are available from Soccer Universe at the following prices:

If you buy this many uniforms	You pay this price for each
1-10	$50.00
11-20	$45.00
21-40	$40.00

a. Your assistant says she sees a way to get one uniform for free. How could she do that? Explain your reasoning and show any graphs, tables, or formulas you used.

10 uniforms cost 10 × 50, or $500 at the 1–10 price. 11 uniforms cost 11 × 45, or $495 at the 11–20 price. So, the 11th uniform is like getting a uniform for free.

Soccerama also sells uniforms. Their base price for one uniform is $50. But discounts for buying more are available. Their chart reads as follows:

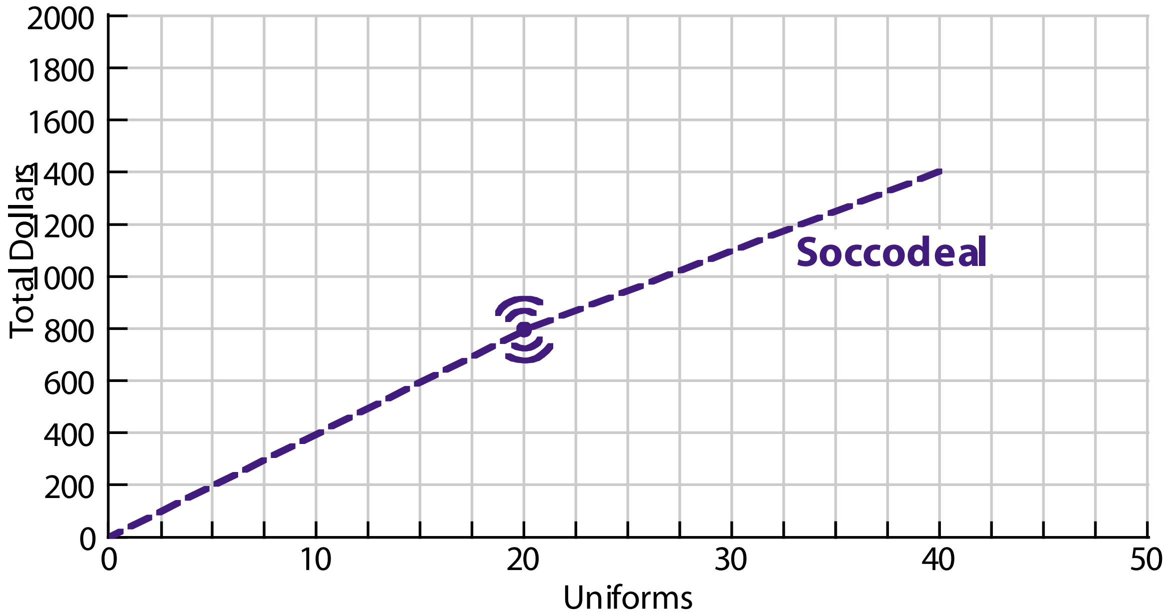
Number of uniforms	Discount on total order
0-15	0%
16 - 25	10%
Over 25	15%

b. Which store—Soccer Universe or Soccerama—would be cheaper if you want to buy 10, 20, or 30 uniforms? Explain your reasoning.

Students may want to make a table as follows:

Number of uniforms	Soccer Universe	Soccerama
10	$500	$500
20	$900	$900
30	$1200	$1275

2. The purchasing department cannot understand the graph from Soccodeal's website shown below.



Write a memo explaining how they should determine the cost of an order for 1 to 40 uniforms.

There are two costs for the uniforms. Between 0 and 20, the uniforms cost $40 each (800/20). Between 20 and 40, the uniforms cost $30 each (600/20).

For example, 30 uniforms cost $1100, because it costs $800 for the first 20 (20 × 40), and $300 for the next 10 (10 × 30). And $800 + $300 = $1100.

You can also estimate by reading the graph. The point on the graph above the 30 on the "Uniforms" axis is about halfway between 1000 and 1200 on the "Total Dollars" axis, thus $1100.

Lesson Plan Day 10

Activities

Manager's Report	Group work
Mathematically Speaking	Whole class discussion

Big idea

"Manager's Report" allows students to report what they have learned within the overall context of the unit. "Mathematically Speaking" helps students connect mathematical vocabulary to the unit's concepts.

Notes

- Each student should do a separate report.
- "Mathematically Speaking" should be teacher led.

Group work

Manager's Report

It has been a busy two weeks, and you have accomplished a lot as the acting soccer team manager. Write a memo to the personnel office director describing what you have done. Highlight one problem you solved using mathematics. Explain why they should hire you as the permanent manager.

Dear Ms Fuentes,

After two weeks as acting manager, I have solved many problems for our soccer team.

List them.

My mathematical skills and understanding have been valuable in solving problems. For example,

Explain what you did with mathematics to solve a problem.

I believe I can help the soccer team as the permanent manager. In addition to problem solving, my strengths include

Fill in your best qualities.

I look forward to hearing from you.

Sincerely,

____

Your name.

Mathematically Speaking

You have investigated some real-life situations and solved problems using mathematics, developing some important mathematical ideas along the way. It is also important to know the mathematical terms that go with these ideas.

KEY IDEA: Proportion

If you can write a formula $y = kx$ where k is a real number other than 0, for two variables x and y, then we can say that

x and y are proportional,

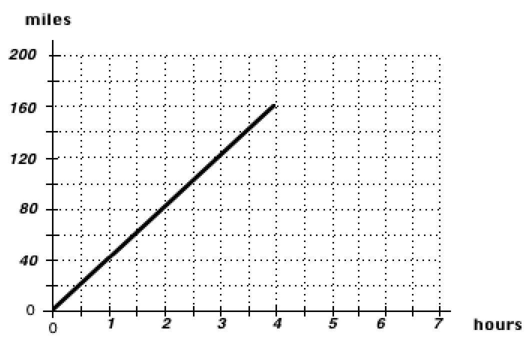
x and y are in proportion,

OR

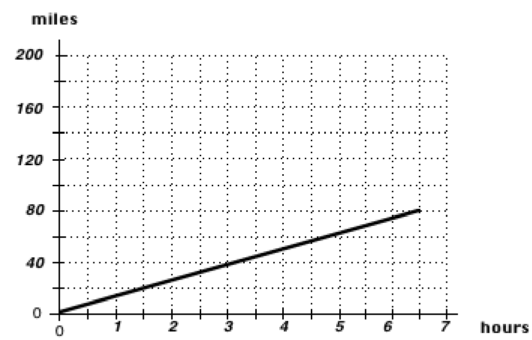
x and y vary proportionally.

These graphs show variables that are in proportion.

a.

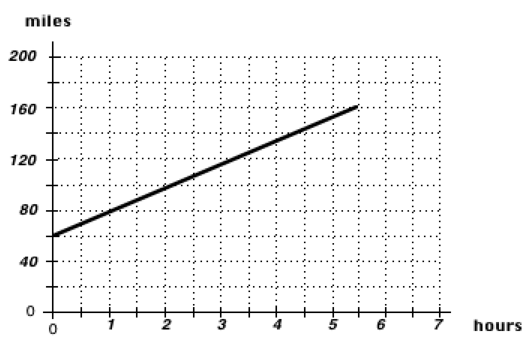


b.

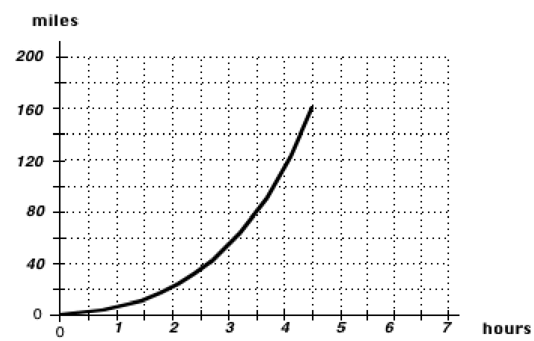


These variables are NOT in proportion.

c.



d.



1. In the graph above, explain why the top two graphs (a and b) show variables in proportion and why the bottom two (c and d) do not.

(Answer on next page.)

Elicit the following:
We know from our work in this unit that (a) shows the graph of an object (vehicle) going 40 miles per hour and that we can write the formula, M = 40H to find the distance for any time. So 40 is the k in y = kx. So the graph shows a proportional relationship.
Similar argument for (b) .
For (c) and (d), first have students try to find formulas.
For (c), show that you can find the slope of the line, but the equation using that slope as k does not work. (Use it to find points, show that these points are not on the line.)
For (d), it will be enough to say that you can't use the slope formula on this curve, the formula will be more complicated.

KEY IDEA: Rate

A rate tells you how many *y* you get for some number of *x*.

A unit rate tells you how many *y* you can get for **one** *x*.

Typically, a rate describes how much of something corresponds with something else: miles per gallon, meters per second. It is often useful to consider how many *y* you get for **one** *x* ; this is called a unit rate. A unit rate tells us—for example—how many miles can be traveled on one gallon; how many meters you can run in one second. This makes it easy to use the rate to think about how many miles you can get from a whole tank of fuel, and how many meters you can cover in 20 seconds.

2. Show different ways to find the unit rate of miles per hour for the top two graphs (a and b) on the previous page. Write the $y = kx$ formula for each graph.

a. **b.**

3. Give examples of unit rates you have used in your life and in this unit.

Photo and Art Credits

Mapquest

US Department of Energy

http://www.fueleconomy.gov

Michael Griffin

Lynne Peck Theis

Managing the Soccer Team: Comments from Jennifer about content, timing and pacing.

What follows is my best advice regarding timing and pacing of the unit. My advice comes from a developer's perspective and is based lessons learned from our pilots of these materials. BUT I have not taught this unit to children myself, so take it all with a grain of salt. In the end, of course, the timing is your call!

- *Time saving advice in italics.*

Day 1

Managing the Soccer Team	Whole class discussion
A Race Day	Whole class discussion
Another Race Day	Group work
Information Quest: How Fast?	Homework

This is a lot to do in one day, but you probably can do most of it.

Managing the Soccer Team should be very quick: it just sets a context—**5 minutes**. Remind students that they should use their common sense through the whole unit, just as if they were doing the manager's job.

A Race Day introduces the conventions of graphing and the motion simulation and connections. This is important. Make sure students understand the conventions. This should be the bulk of the class. **25 minutes**.

Another Race Day presents problems students can do in groups or on their own. It builds on the whole class lesson. Students watch a race, predict its graph, verify their prediction and then interpret points on the graph. *You can talk them through 3b quickly if needed*. **15 minutes.**

Info Quest is homework. Tell students they don't have to find all the speeds, just as many as they can.

Day 2

Isabella Improves	Whole class discussion
Faster Than Max	Group work
Practice Runs	Homework

Isabella Improves helps students refine their understanding of the graphs and their meanings by looking at a pattern formed over several dashes. The discussion should help them establish, informally, the meaning of the steepness of the graph. **25 minutes.**

Faster Than Max should then be done in groups. **10 minutes** for part 1 and **10 minutes** for part 2 where they exchange books. *Skip part 2 if you run out of time*.

Practice Runs can be done at home. You can assign just a few if it seems like too many problems.

Day 3

Run, Jace, Run	Whole class discussion
Run, Jace, Run, Revisited	Group work/Homework

Run, Jace, Run is a critical lesson. Students write formulas and connect all three: formulas, tables and graphs. This may take **45-50 minutes.**

Run, Jace, Run Revisited is homework. It is matching, not too hard. *Send it home*.

Day 4

Back at the Office	Whole class discussion

Motion is not the only situation that can be modeled in MathWorlds. These problems involving purchasing contrast number of items with total cost. The unit rate involved is the price. Students get opportunities to solve problems. You may want to assign some as group work. *You can skip number 3 if you run short of time*. **45-50 minutes**

Day 5

Slope and Rate	Whole class discussion

Time to really define slope. Take your time using the slope triangles to help students understand this property of the line. **45-50 minutes**. *You can assign number 3 as homework or even extra credit.*

Day 6

On the Road	Whole class discussion
Road Trips	Group work

In **On the Road**, things get more complicated: these vehicles change rates. So now we have two objects moving, but each at more than one rate. Help students interpret these graphs with stories. **20 minutes.**

In **Road Trips**, students draw and interpret graphs. *Problem 3 can be homework as it does not require a computer*. **20 minutes**.

Day 7

Graphs and Motion	Group work

Graphs and Motion provides practice on interpreting or creating graphs. You can *assign as many problems as students can do in one class period (for example: 1a, b; 2 a.)* Item 3 is important and you may want to stop and go over it at the end of the period. **45-50 minutes**

Day 8

Salary Negotiations	Whole class discussion
Summer Job Advice	Group work
All About MPG	Homework

Salary Negotiations and **Summer Job Advice** present problem solving opportunities from which *you can pick and choose*. **45-50 minutes**

All About MPG is for homework to help students get (or recall) the basic idea of "miles per gallon"—rate of fuel consumption. This is in preparation for Day 8.

Day 9

How Far On How Much? MPG	Whole class discussion
Suiting Up	Group work

You could skip any of these if the unit is running too long. They are problem-solving opportunities in different contexts.

Day 10

Manager’s Report	Group work
Mathematically Speaking	Whole class discussion

The **Manager’s Report** *can be assigned as homework* if you don’t want to take class time. Do NOT skip **Mathematically Speaking**; it is an important discussion but will probably take at most **30 minutes.**